

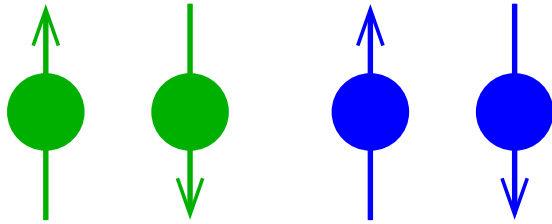
α —Particle Condensation in Nuclear systems

A. Tohsaki, H. Horiuchi, G. Röpke, P. Sch.
T. Yamada and Y. Funaki

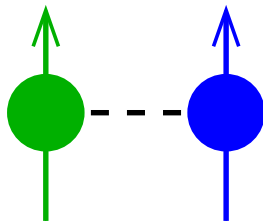
November 29, 2005

- α -condensation in ∞ matter
- ^8Be and Hoyle state in $^{12}\text{C}^*$
- α -condensate wave function
- Effective GPE for $n.\alpha$ condensate
- Conclusions, outlook

Clusters important aspect and richness of nuclear systems due to 4 Fermions :

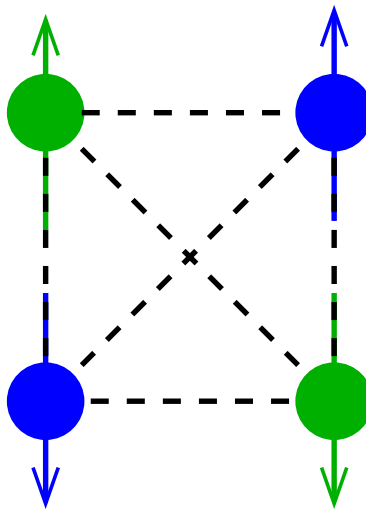


Dimer :

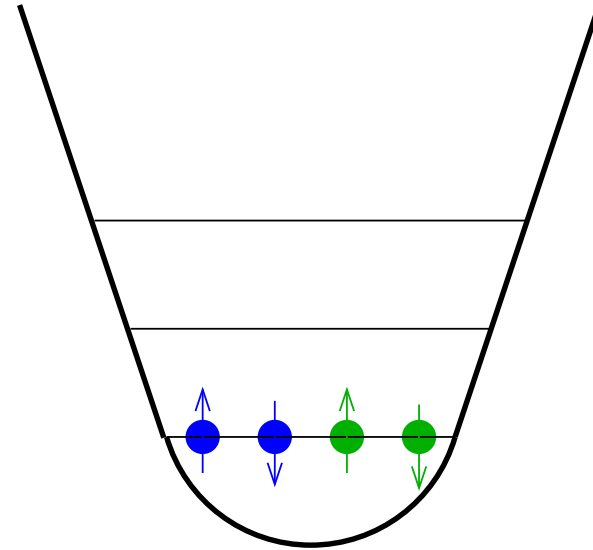


$$\frac{E}{A} = 1 \text{ MeV}$$

Quartet :



$$\frac{E}{A} = 7 \text{ MeV}, \quad E^* = 20 \text{ MeV}$$



Proposal :

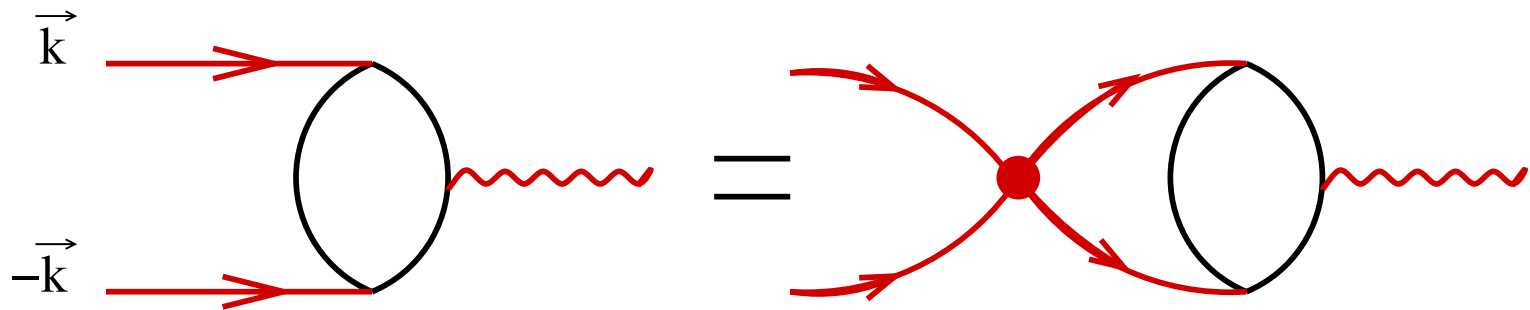
Trapping of 4 different species of Fermionic atoms.

Infinite matter :

- Pair Condensation (nn or pn) $\epsilon_1 = \frac{p_1^2}{2m}$
- Thouless criterion for T_c : $f_1 = \frac{1}{1 + e^{\frac{(\epsilon_1 - \mu)}{T}}}$

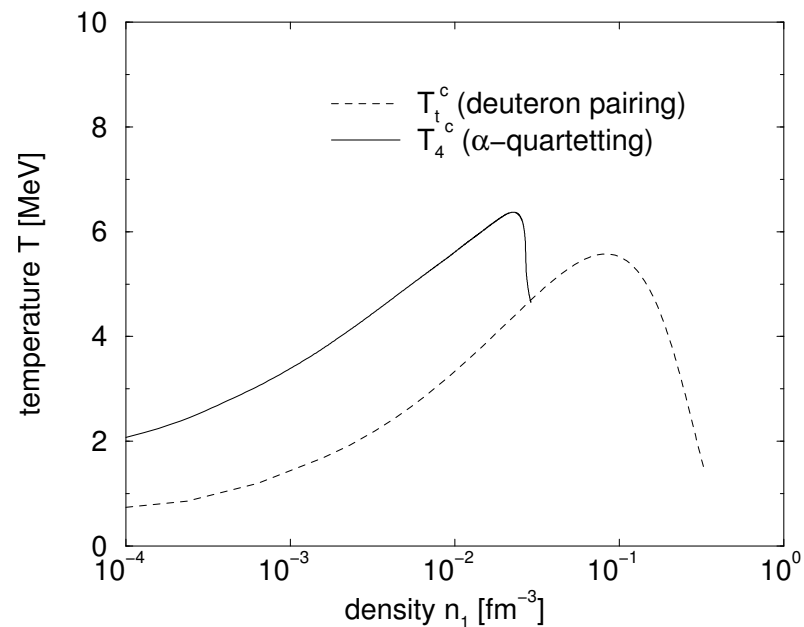
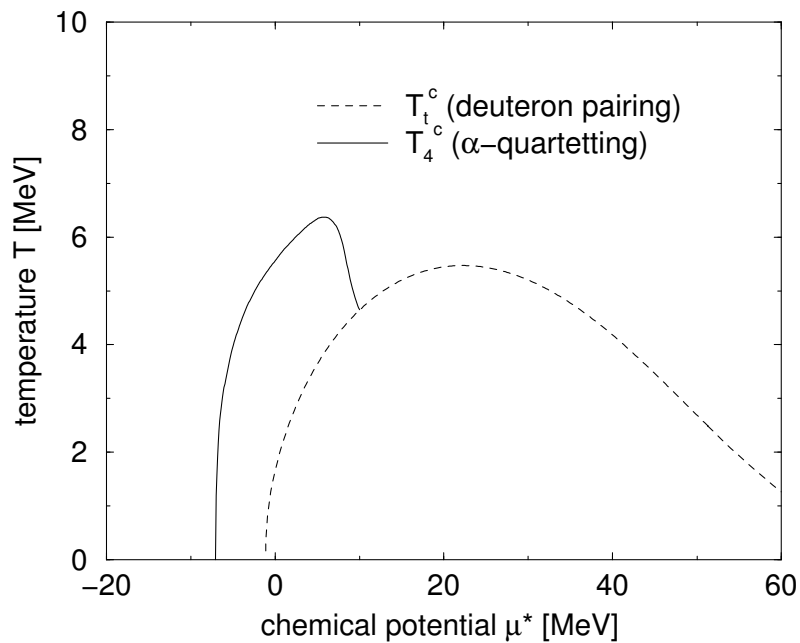
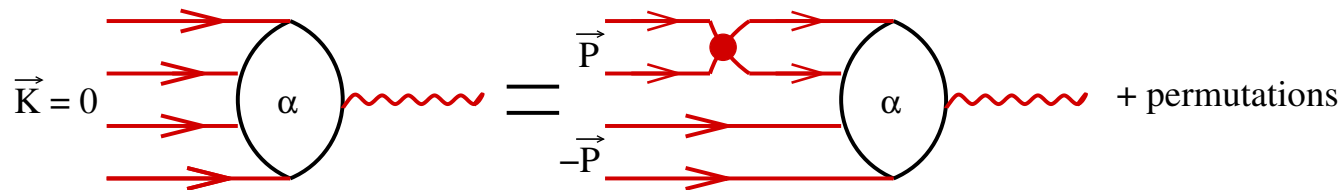
$$(2\mu - \epsilon_1 - \epsilon_2) \psi_{12} = (1 - f_1 - f_2) \sum_{1'2'} v_{121'2'} \psi_{1'2'}$$

μ chemical-potential and f_1, f_2 Fermi-Dirac at $T = T_c$.

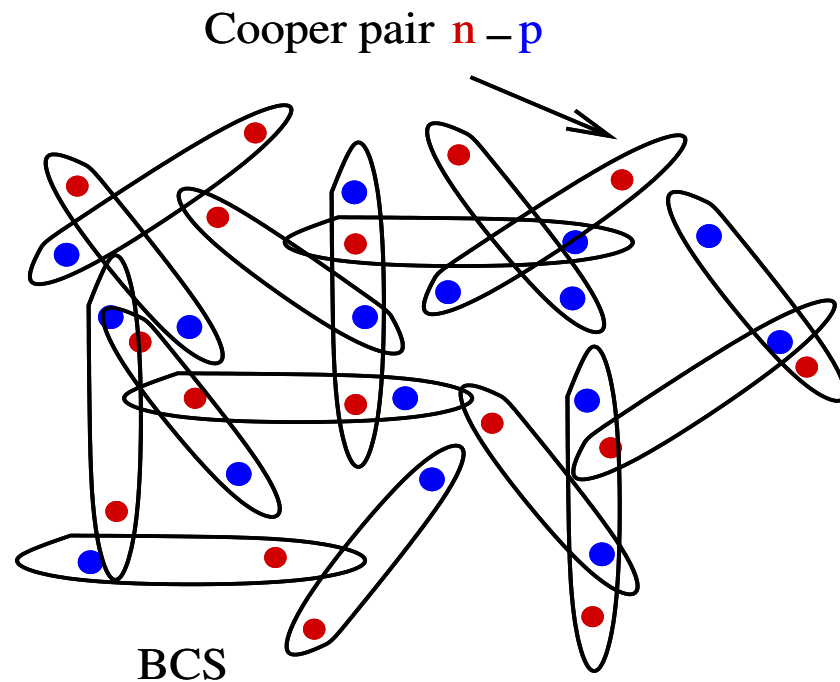


- α -Particle Condensation : G. Röpke, M. Beyer

$$(4\mu - \epsilon_1 - \epsilon_2 - \epsilon_3 - \epsilon_4) \psi_{1234} = (1 - f_1 - f_2) \sum_{1'2'} v_{121'2'} \psi_{1'2'34} + \text{permutations}$$



α -Condensation only at very low density !



High Density

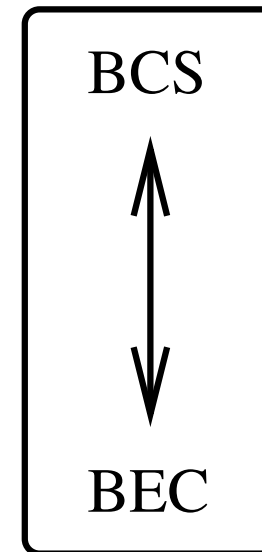
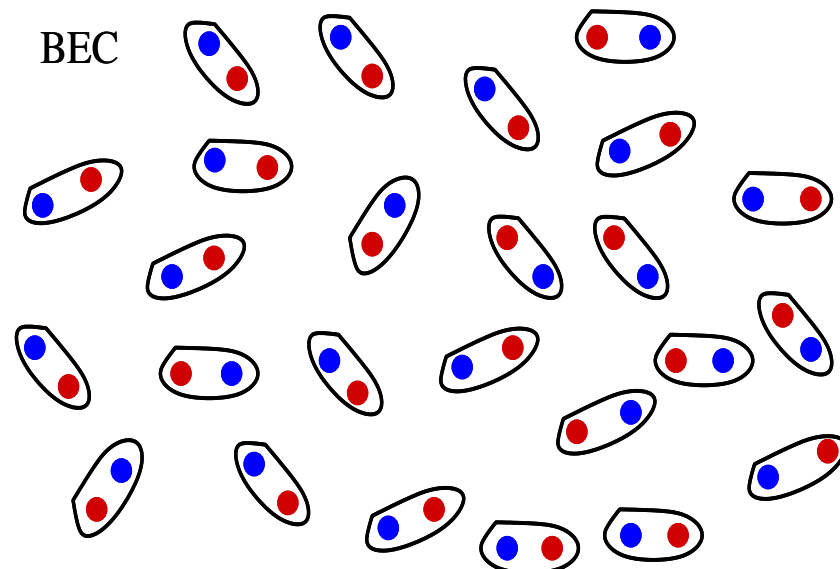
$n-p$ Cooper pairs

Strongly overlapping

not Bosons

Low density :

smooth transition



gas of Deuterons

$\sim$ Bosons

Finite nuclei ?

Exact ${}^8\text{Be}$:

Density : $\frac{\rho_0}{3}$

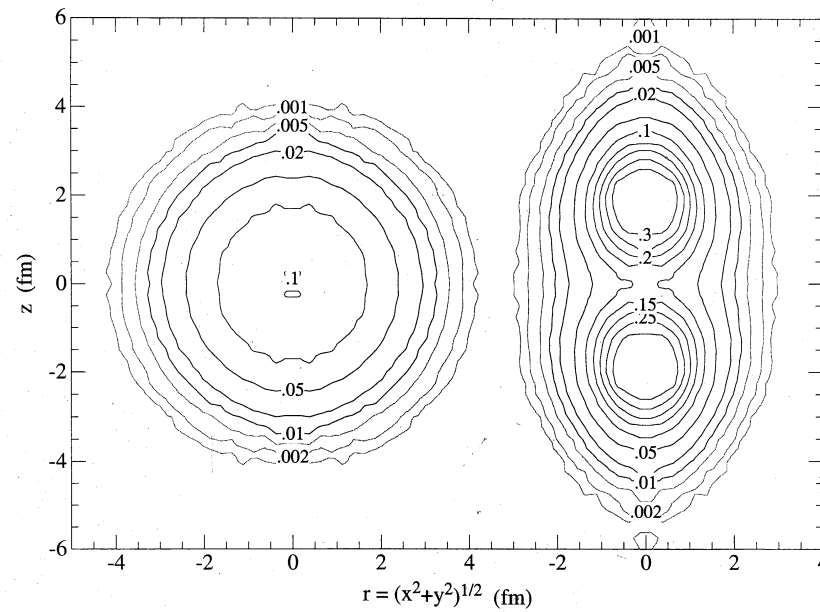
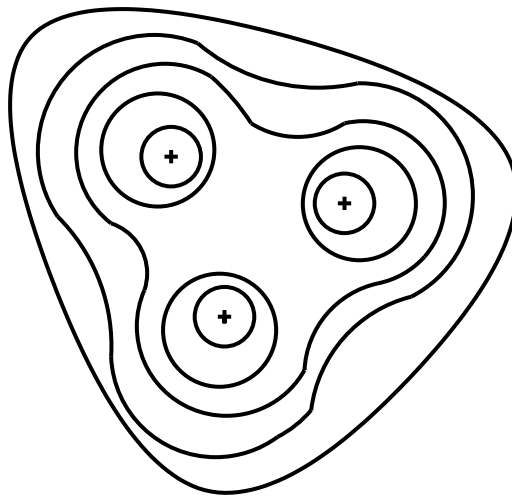


Fig. 15 (Wiringa, et al.)

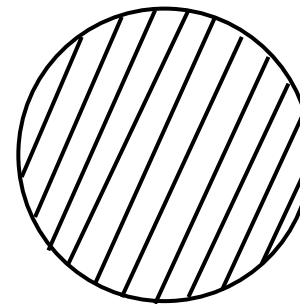
3 rd α -particle



V

collapse
 $\Rightarrow$

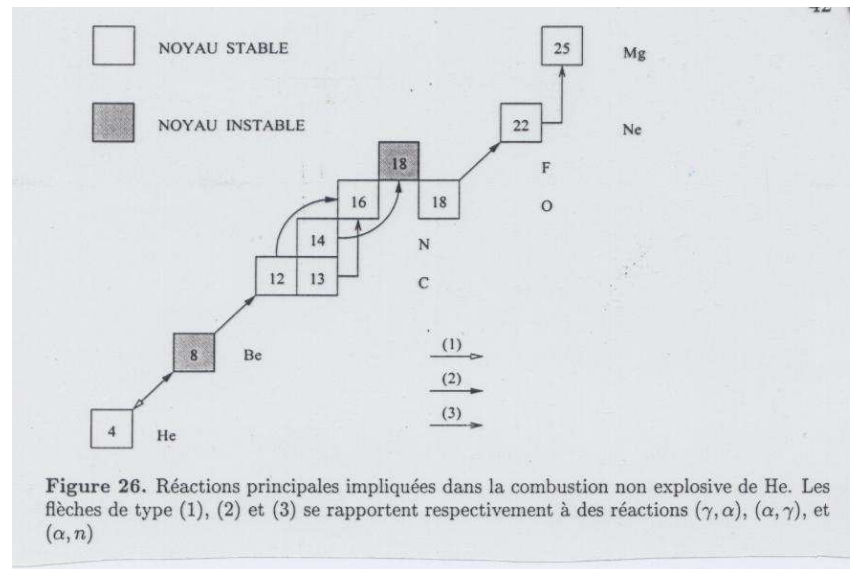
Fermi gas



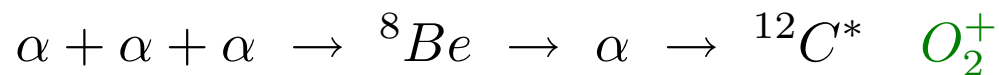
compact ground state $V/3$

${}^{12}\text{C}$

Does a dilute 3α $^{12}\text{C}^*$ state exist ?
 Similar to $^8\text{Be} + \alpha$?



At $T = 10^8 \text{K}$ helium burning
 thermal equilibrium



7 O_2^+ : dilute 3α state hypothesis !

feature article

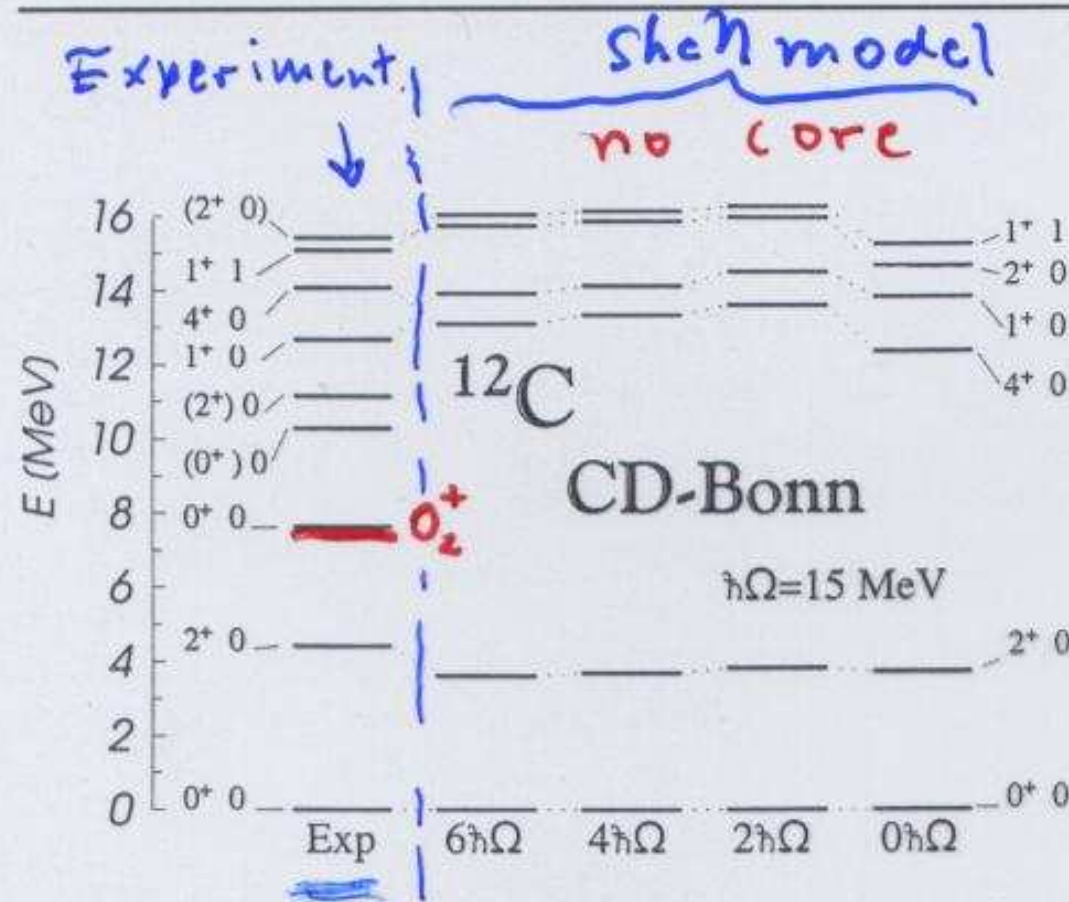
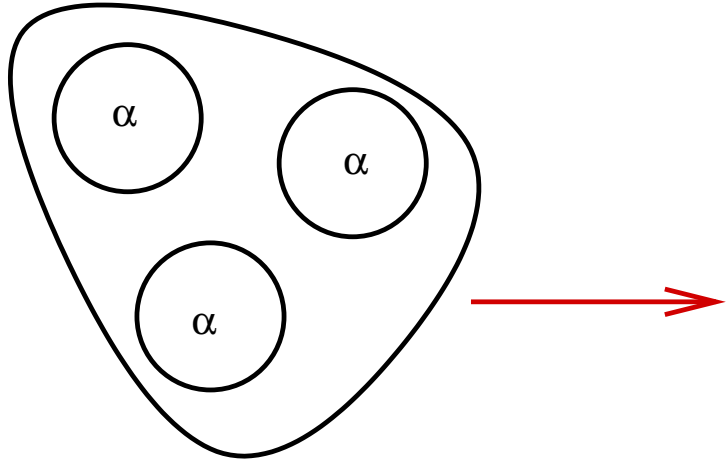


Figure 4. Experimental and NCSM excitation spectra for ^{12}C

it seems impossible to get Hoyle state from shell model calculation !

45 MeV B. Barret

If O_2^+ in ^{12}C dilute α – state

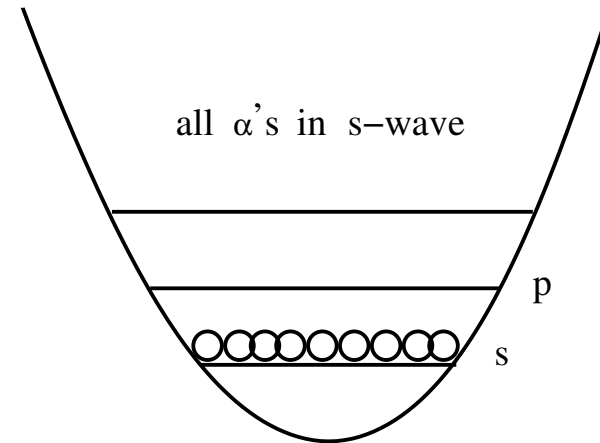
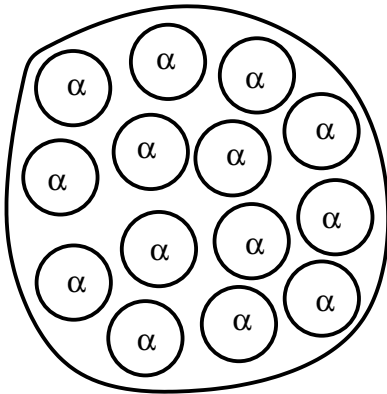


then α -condensate

infinite matter $\rho_{\text{crit}} \sim \frac{\rho_0}{3}$

Conjecture

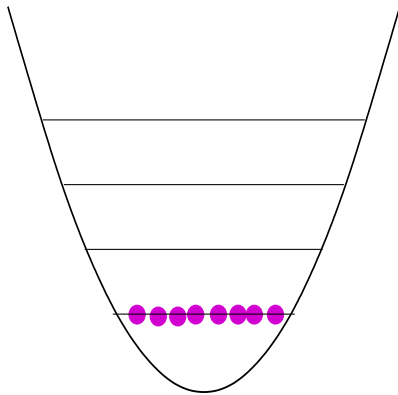
all $n.\alpha$ nuclei possess excited $n\alpha$ condensed state



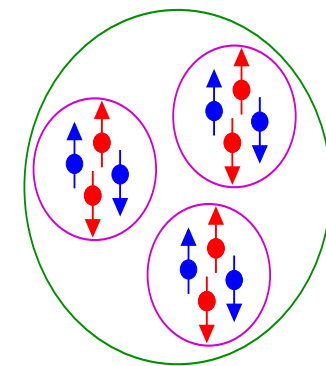
Analogy with atoms in traps ! $\rho(r) = N|\phi_0(r)|^2$

$N = 10^6$

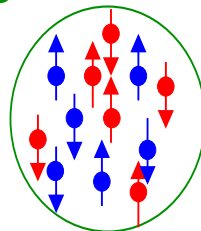
Bosons



Back to nuclei



^{12}C



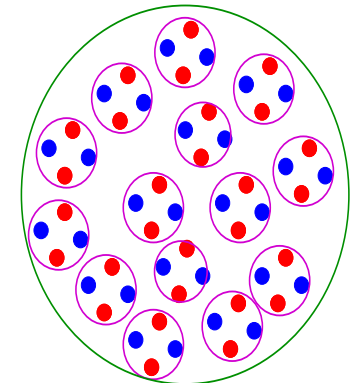
— 0_2^+ 7,65 MeV

— 0_1^+

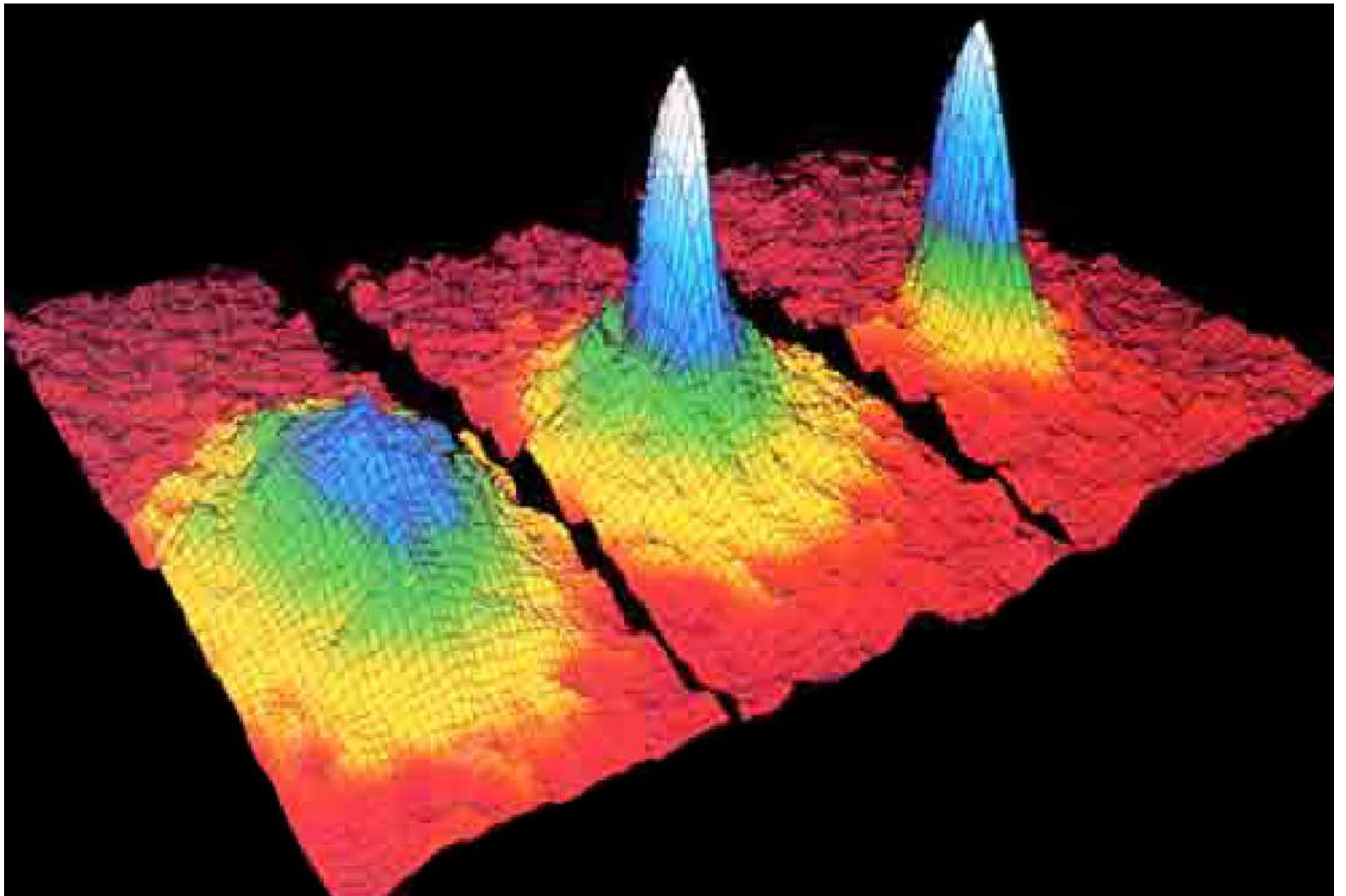
● proton
● neutron
○ alpha

many α 's

→ condensate



strong cluster
phenomena in
lighter nuclei



Theoretical Description

Ideal Bose condensate : $|0\rangle = b_0^\dagger b_0^\dagger \cdots b_0^\dagger |vac\rangle$

α -particle condensate : $|\Phi_{\alpha C}\rangle = C_\alpha^\dagger C_\alpha^\dagger \cdots C_\alpha^\dagger |vac\rangle$

In r -space :

$$\langle \vec{r}_1, \vec{r}_2, \cdots, \vec{r}_{4n} | \Phi_{\alpha C} \rangle = \mathcal{A} \left\{ \Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) \Phi(\vec{r}_5, \vec{r}_6, \vec{r}_7, \vec{r}_8) \cdots \Phi(\vec{r}_{4n-3}, \vec{r}_{4n-2}, \vec{r}_{4n-1}, \vec{r}_{4n}) \right\}$$

In comparison with pairing :

$$\langle \vec{r}_1, \vec{r}_2, \cdots | \text{BCS} \rangle = \mathcal{A} \left\{ \Phi(\vec{r}_1, \vec{r}_2) \Phi(\vec{r}_3, \vec{r}_4) \cdots \right\}$$

Variational ansatz for $\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4)$:

$$\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = e^{-\frac{2}{B^2} \vec{R}^2} \phi_\alpha(\vec{r}_i - \vec{r}_j)$$

Center of mass : $\vec{R} = \frac{1}{4}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4)$

Intrinsic α -wave function :

$$\phi_{\alpha}(\vec{r}_i - \vec{r}_j) = e^{-\frac{1}{8b^2}\{(\vec{r}_4 - \vec{r}_1)^2 + (\vec{r}_4 - \vec{r}_2)^2 + (\vec{r}_4 - \vec{r}_3)^2 + \dots\}}$$

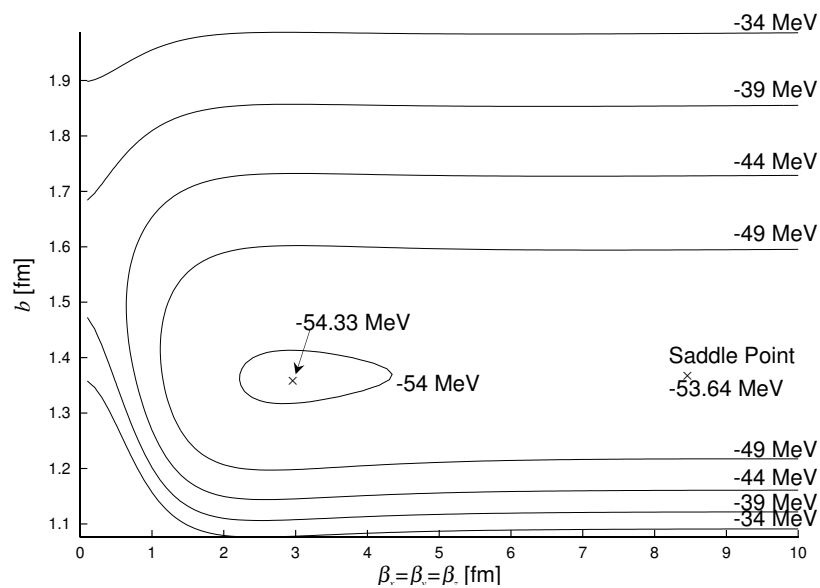
Two variational parameters : B, b

Two limits : $B = b$ $|\Phi_{\alpha C}\rangle =$ Slater determinant

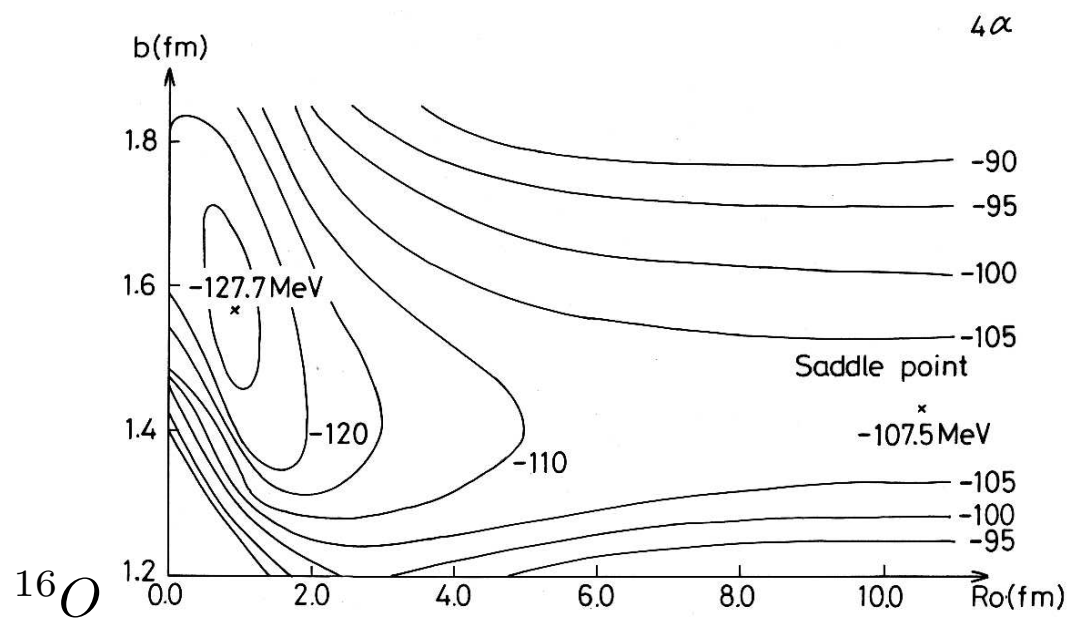
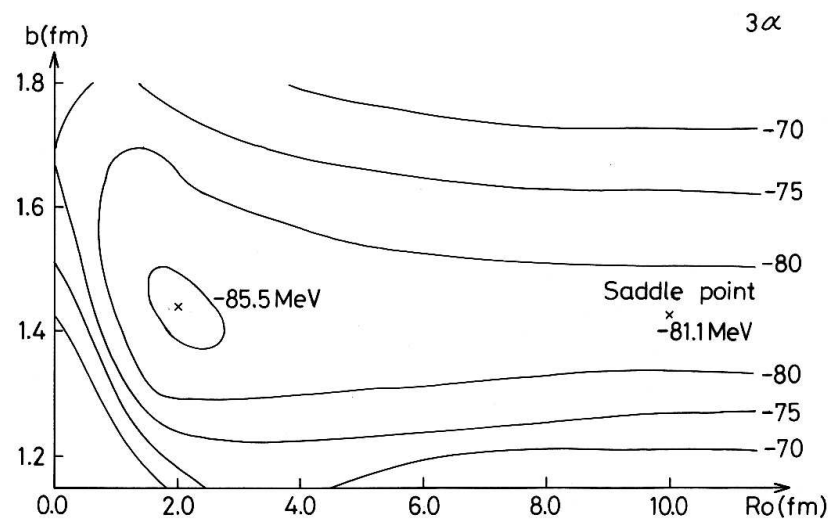
$B \gg b$ $|\Phi_{\alpha C}\rangle =$ gas of independent α -particles

Two dimensional surface : $E(B, b) = \frac{\langle \Phi_{\alpha C} | H | \Phi_{\alpha C} \rangle}{\langle \Phi_{\alpha C} | \Phi_{\alpha C} \rangle}$

8B



${}^{12}C$



Hamiltonian :

$$H = \underset{\text{Kin. energy}}{T} + \underset{\text{Gaussien}}{V_{N-N}} + \underset{\text{Coulomb}}{V_C} + \underset{\text{Gaussian}}{V_{N-N-N}}$$

Quantization of energy surface $E(B, b)$:

Force : A. Tohsaki ~ 1990 no adjustable parameters !

Hill-Wheeler : $|\psi\rangle = \sum_B f_B |\Phi_{\alpha C}(B)\rangle$

Without adjustable parameters :

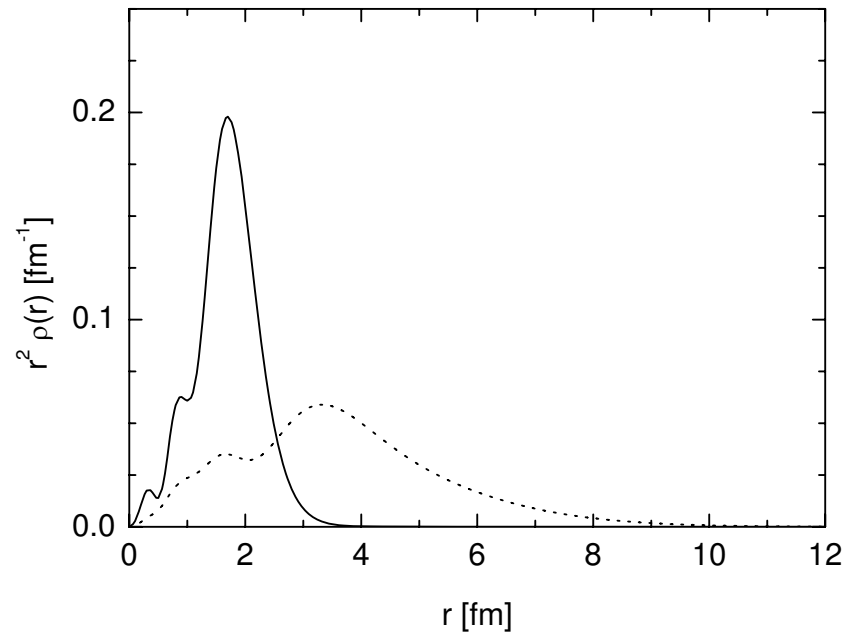
$$\begin{array}{lcl} {}^{12}\text{C} : & (E_{O_2^+} - E_{3\alpha}) & = \text{Theory} + 0.50 \text{ MeV} \\ & & \text{Exp.} + 0.38 \text{ MeV} \\ {}^{16}\text{O} : & (E_{O_5^+} - E_{4\alpha}) & = \text{Theory} - 0.70 \text{ MeV} \\ & & \text{Exp.} - 0.44 \text{ MeV} \end{array}$$

r.m.s. of O_2^+ in ^{12}C
groundstate

3.83 fm
2.40 fm

$$\frac{V_{O_2^+}}{V_{O_1^+}} \sim 3 - 4$$

Density :

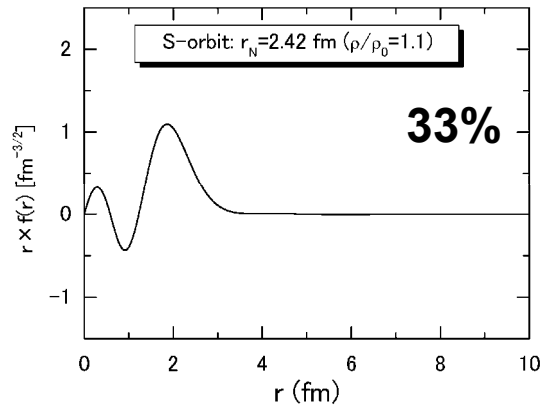


	<i>Exp.</i>	<i>Cal.</i>
$M(O_2^+ \rightarrow O_1^+)$	$5.4 \pm 0.2 fm^2$	6.7
$R_{rms}(O_1^+)$	2.43 fm	2.40
$R_{rms}(O_2^+)$		3.47

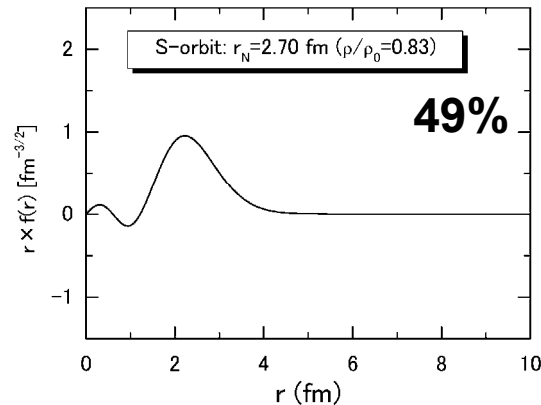
Radial behavior of S-wave α orbit vs. R_{rms}

$$R_{\text{rms}} = 2.43 \text{ fm} \rightarrow 4.84 \text{ fm} \quad (\rho/\rho_0 = 1.1 \rightarrow 0.14)$$

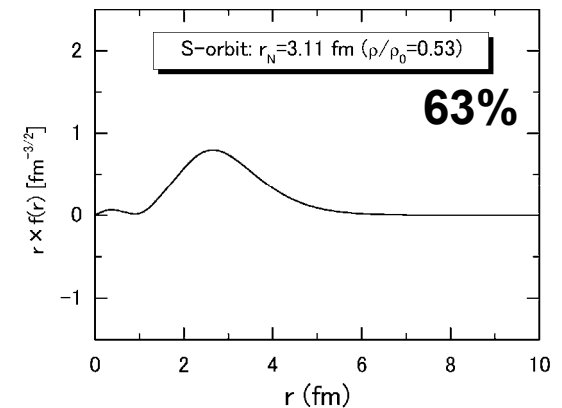
$R_{\text{rms}} = 2.43 \text{ fm} \quad (\rho/\rho_0 = 1.1)$



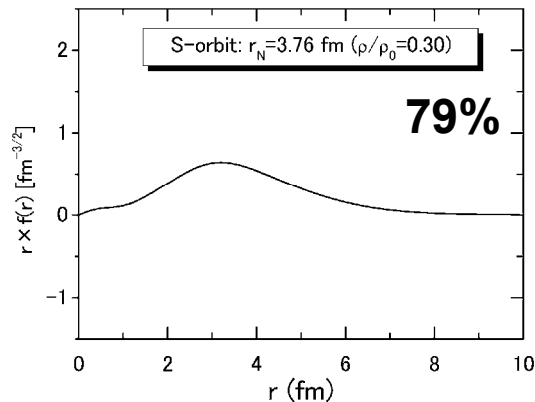
$R_{\text{rms}} = 2.70 \text{ fm} \quad (\rho/\rho_0 = 0.83)$



$R_{\text{rms}} = 3.11 \text{ fm} \quad (\rho/\rho_0 = 0.53)$



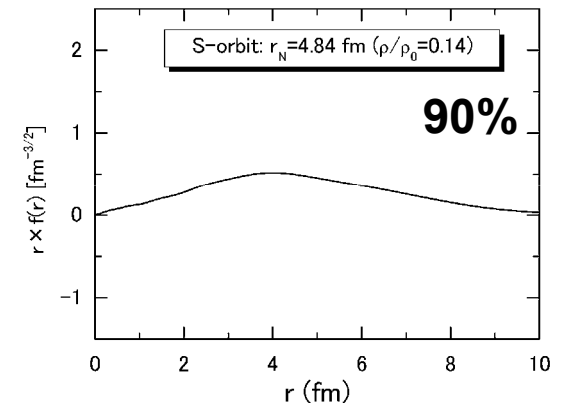
$R_{\text{rms}} = 3.76 \text{ fm} \quad (\rho/\rho_0 = 0.30)$

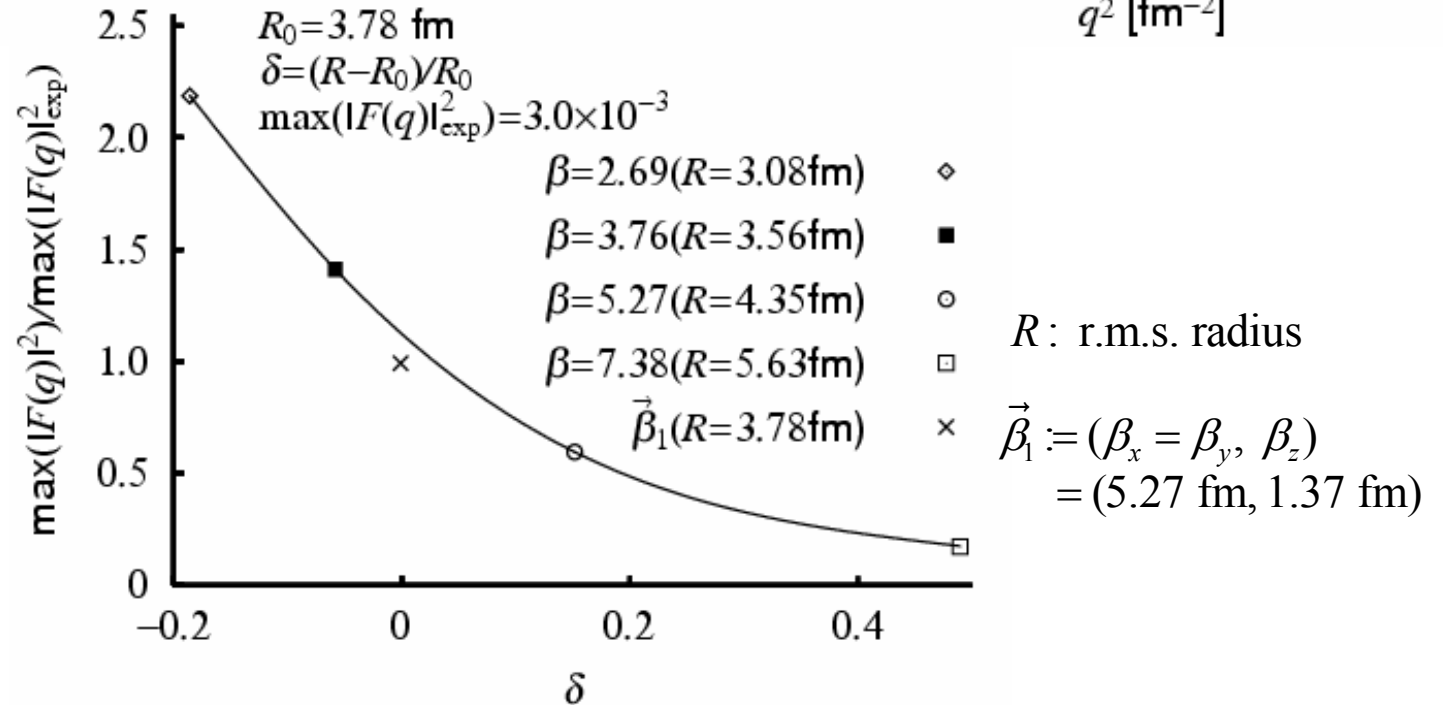
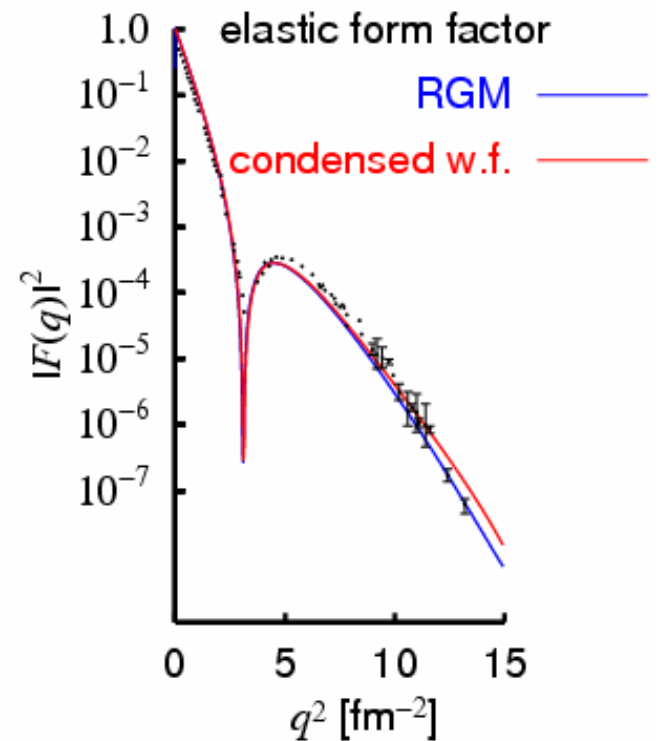
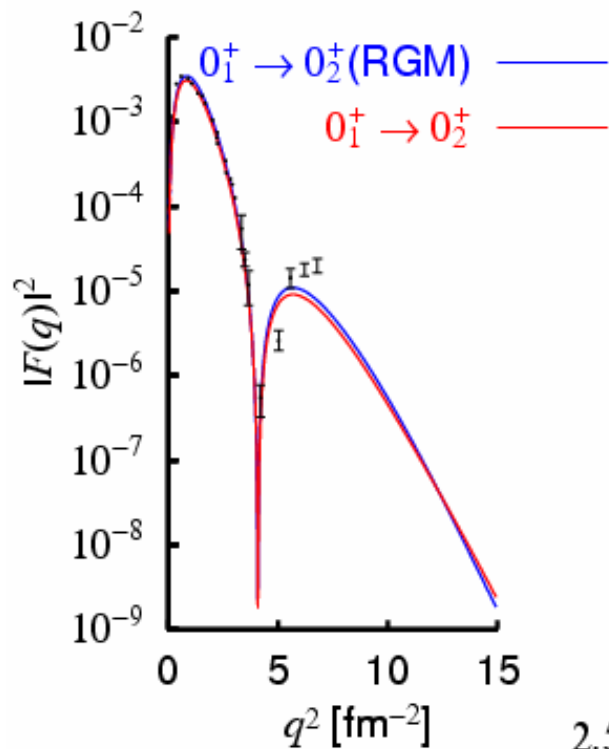


Increasing R_{rms} ,
we see smooth change
from **2S** to **0S** orbit.

Yamada & Schuck, EPJA 2005

$R_{\text{rms}} = 4.84 \text{ fm} \quad (\rho/\rho_0 = 0.14)$



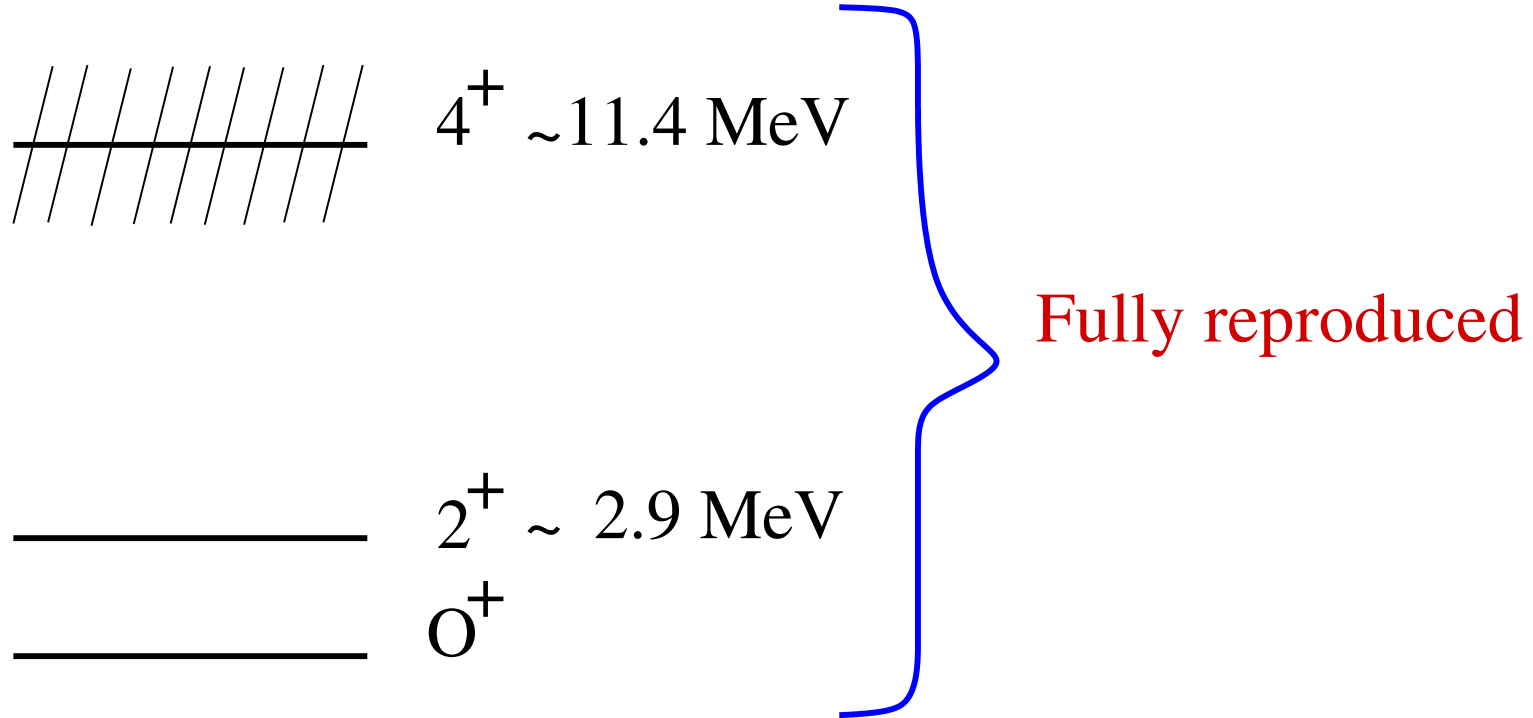


Some more numbers :

		Theory	Exp.
$^{12}\text{C} :$	O_1^+	-89.52	-92.16
	O_2^+	-81.79	-84.51
		7.73	7.65

	^8Be	^{12}C	^{16}O	^{20}Ne	
Threshold	2α	3α	4α	5α	
states	O_1^+	O_2^+	O_3^+	O_4^+	
$E - E_{\text{thresh.}}$	-0.17	0.50	-0.7	1.8	] theory
	O_1^+	O_2^+	O_5^+	?	
	0.09	0.38	-0.44	?	] experimental

Spectrum of ^8Be :



^{12}C : Second excited 2^+ : 2_2^+

It has been discovered recently by Itoh *et al.*

2.6 MeV above 3 α threshold

Width $\sim 1 \text{ MeV}$: resonance in continuum

Theory : We start with deformed α condensate state :

$$\Phi_{n\alpha} \propto \mathcal{A} \prod_{i=1}^n \exp \left\{ -\frac{2 X_{ix}^2}{B_x^2} - \frac{2 X_{iy}^2}{B_y^2} - \frac{2 X_{iz}^2}{B_z^2} \right\} \Phi_{\alpha_i}$$

Then projection on good angular momentum

Then Hill Wheeler or GCM

For width : ACCC method

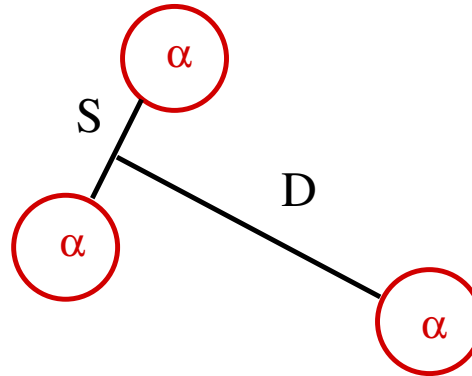
	Position	Width
<u>Experiment :</u>	$2.6 \pm 0.3 \text{ MeV}$	$1.0 \pm 0.3 \text{ MeV}$
<u>Theory :</u>	2.1 MeV	0.64 MeV

With in error bars !

$$\text{RMS : } \frac{V_{2_2^+}}{V_{O_1^+}} \sim 8 \quad !!$$

α – halo !

Internal structure :



Extremely dilute 3α state

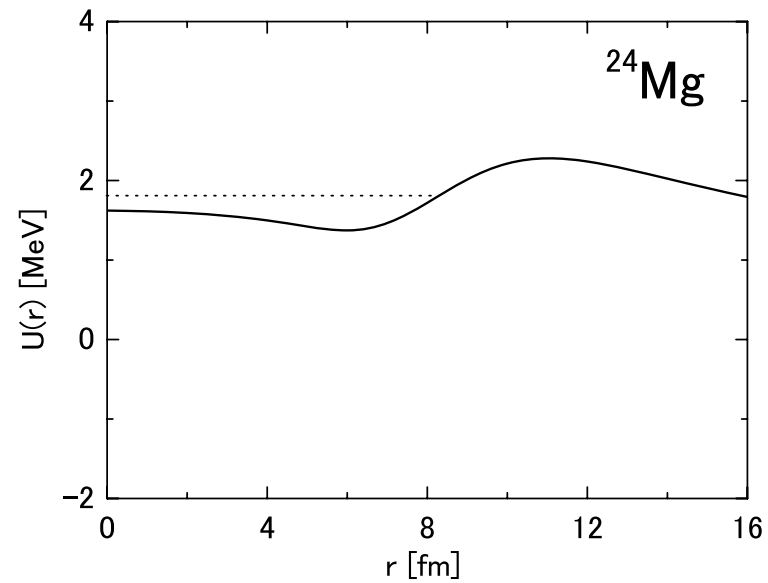
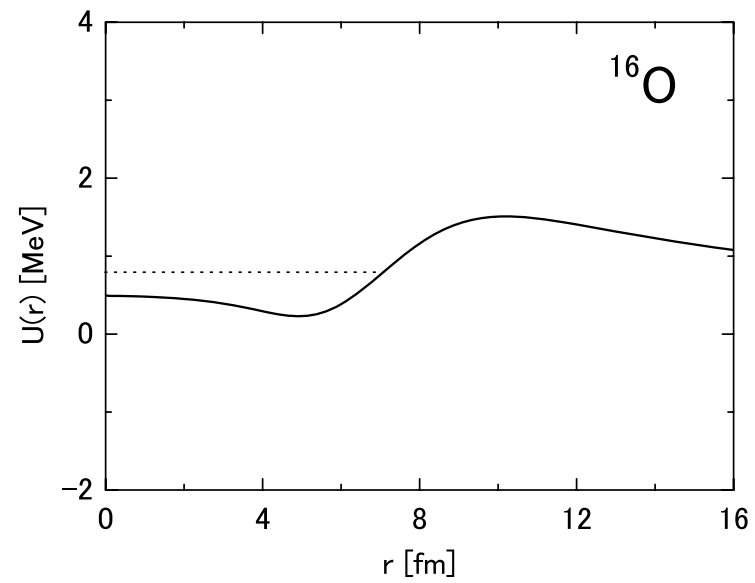
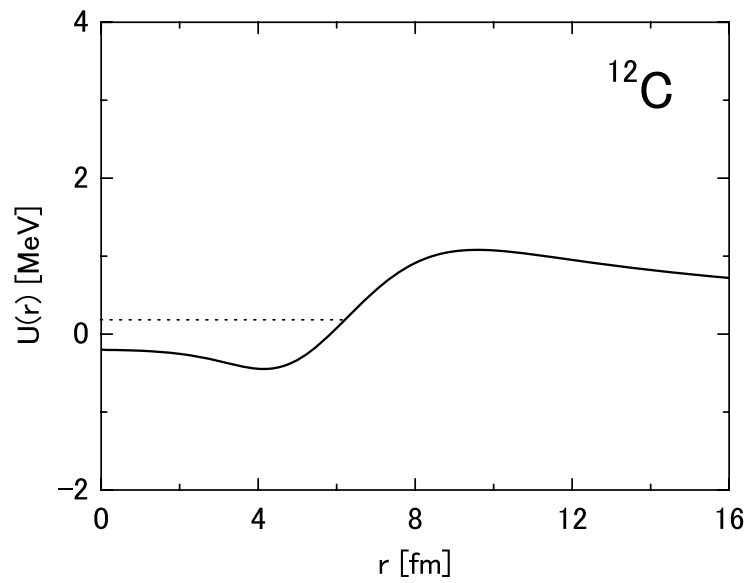
Suggests a pure Boson picture

$$|\phi_0\rangle = b_0^+ b_0^+ b_0^+ \dots |vac\rangle$$

Hartree – Fock (Gross Pitaevsky eq)
for ideal bosons (α' s) :

$$\left[-\frac{\hbar^2}{2m_\alpha} \Delta + N \int d^3r' v(\vec{r} - \vec{r}') |\phi_0(\vec{r}')|^2 \right] \phi_0(\vec{r}) = \epsilon_0 \phi_0(\vec{r})$$

effective $\alpha - \alpha$ + Coulomb
T. Yamada

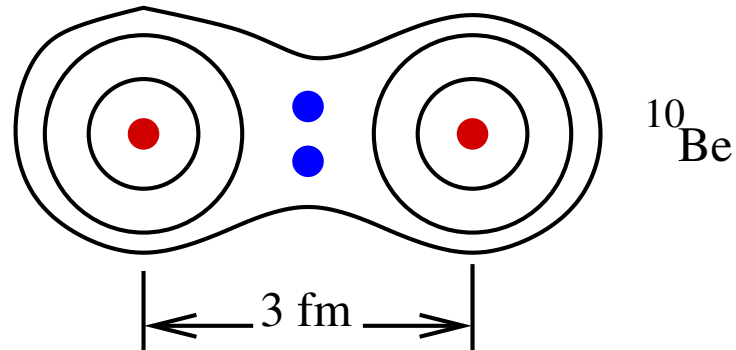
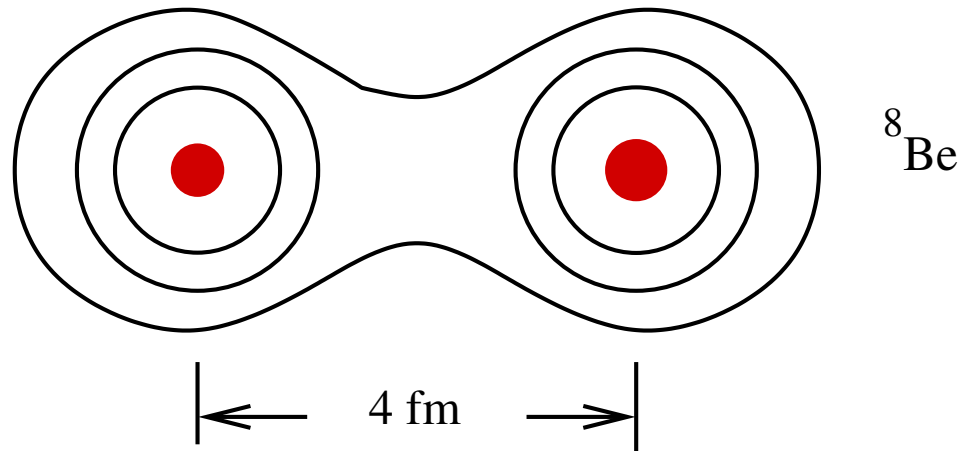


Estimate for maximum number

$$N_{limit}^{\alpha} \simeq 10 \quad \Rightarrow \quad {}^{40}\text{Ca}^{**}$$

BUT

Some neutrons can stabilise
 ^8Be unbound, ^9Be bound (2.5 MeV)
 ^{10}Be strongly bound !
May be 20 - 30 α 's
possible !



Boson occupancy :

α -particle density matrix :

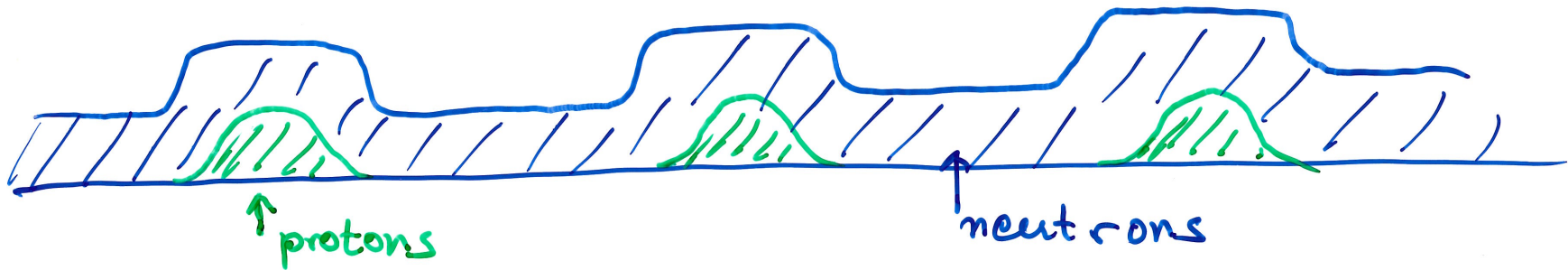
$$\rho_{\alpha}(\vec{R}, \vec{R}'), \quad \vec{R} : \text{c.m. of } \alpha$$

Diagonalization :

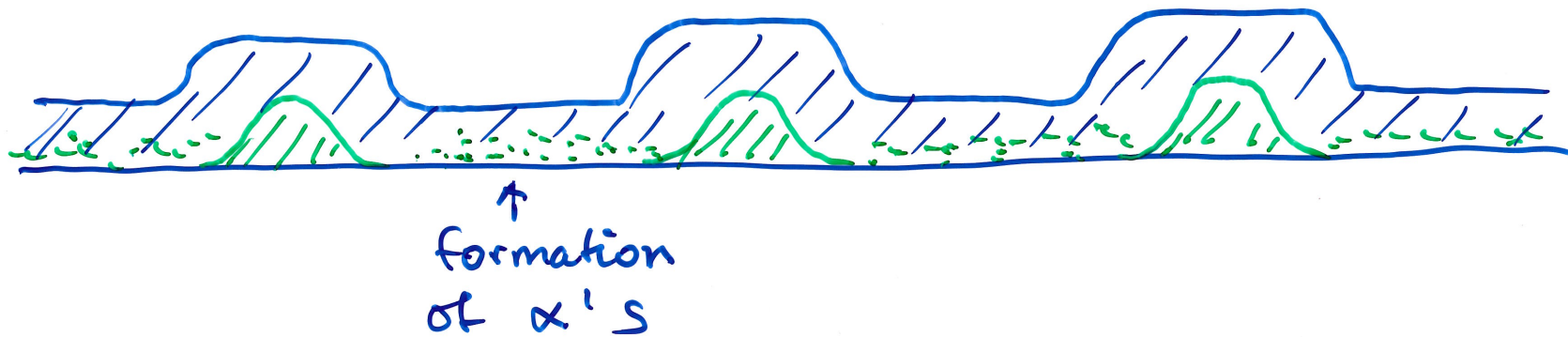
$${}^{12}\text{C} : \quad O_2^+ \quad 70\% \quad \text{S-wave occupancy}$$

Alpha particle condensation in crust of neutron stars?

$T=0$



$T>0$



Conclusions

- Strong indications that extended α -condensates exist, α -condensate-halos
- Adding some neutron glue, condensates may become very large : **ten's of α 's !** Highly excited but long life time !
- **Indra : $^{36}\text{Ar} + ^{36}\text{Ni} \Rightarrow 9\alpha$**
(M.F. Rivet, B. Borderie)
- α -energies $\rightarrow$ wavefcts — Chimera
- Why α , not ^{16}O , ^{40}Ca
 α very inert and compact, almost ideal Boson, 20 MeV first excitation
May be $^{48}\text{Cr} \rightarrow 3 \times ^{16}\text{O}$
- Ultimate question :
How to prove α -particle superfluidity ?

- Eventually important in nuclear astrophysics : collapsing massive stars
- Nuclei immersed in a gas of neutrons + some protons at finite $T \rightarrow$ Can lead to gas of α -particles
- may be α -particle condensate
- change of equation of state
- change of neutrino absorption
- influence on collapse scenario

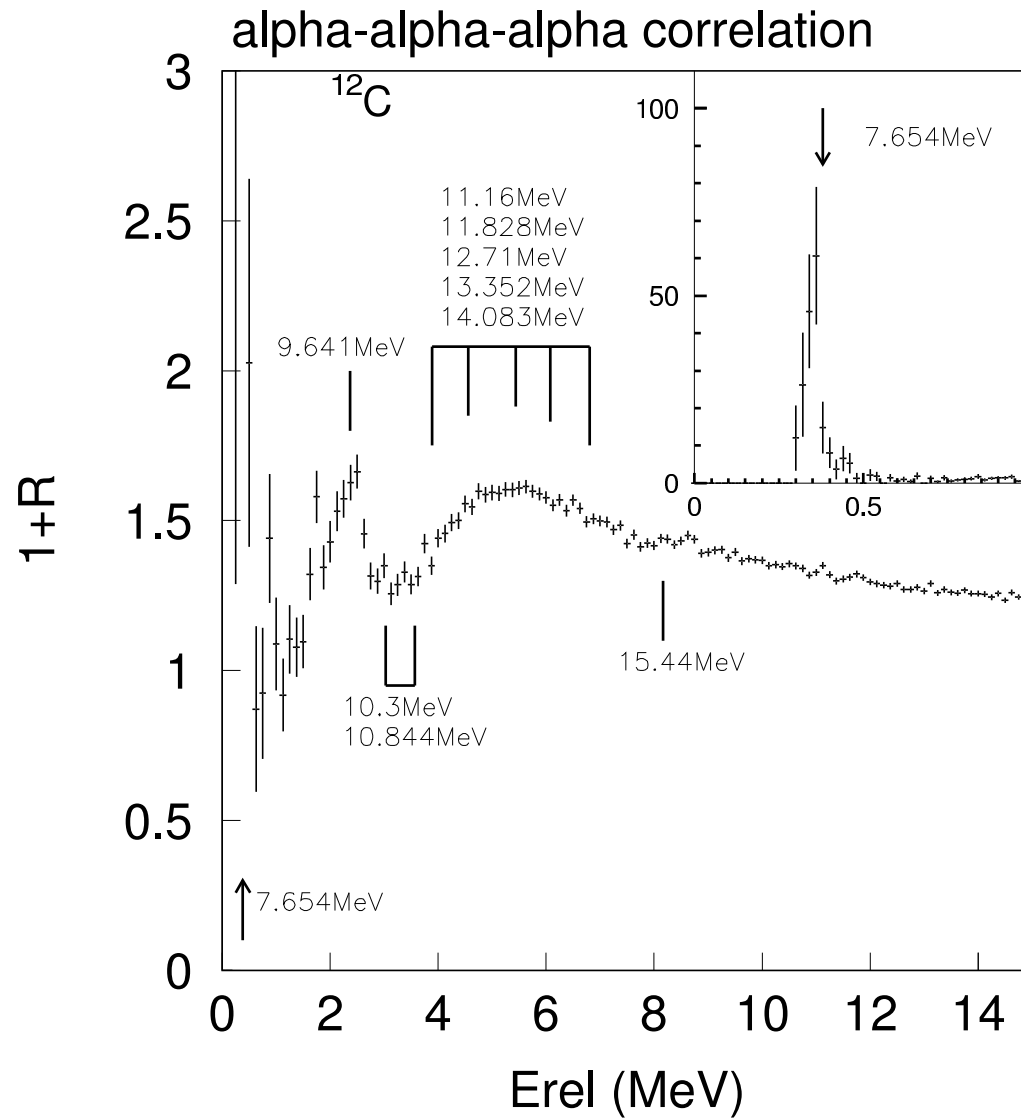
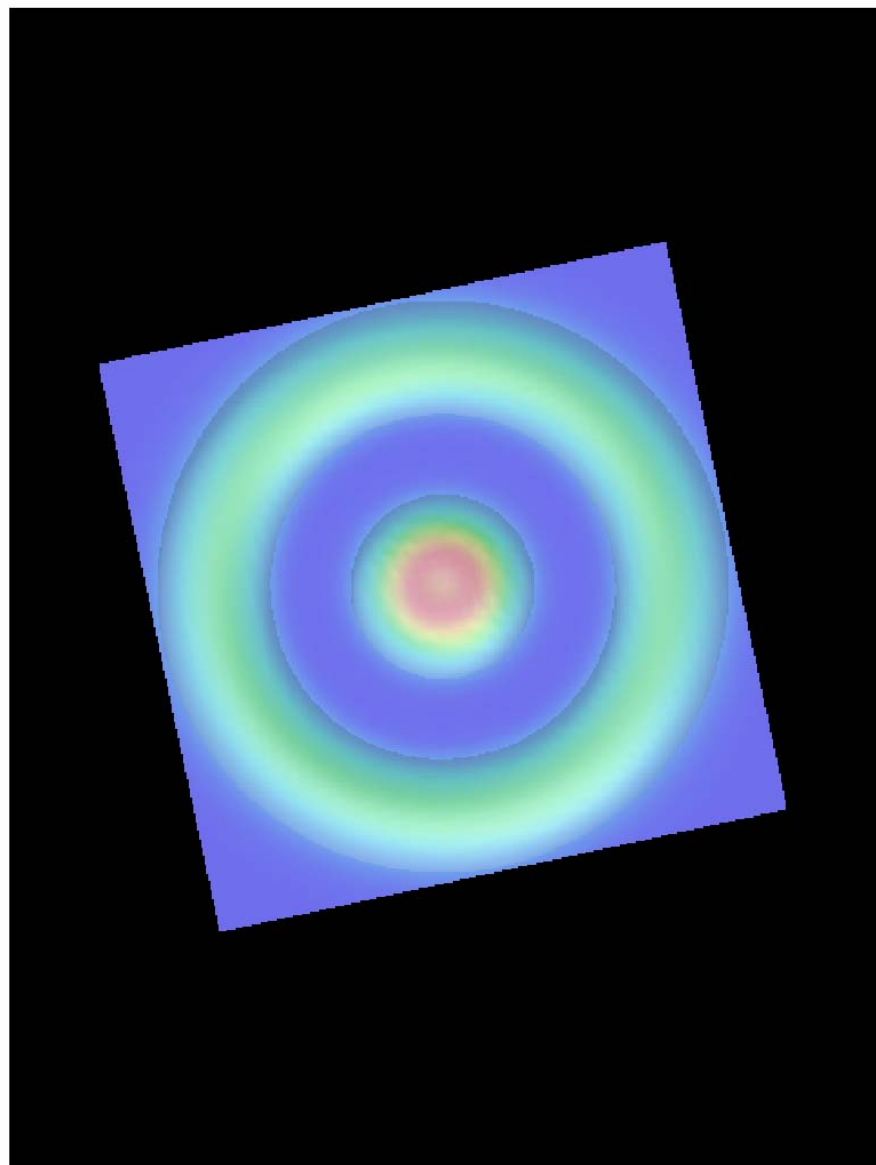
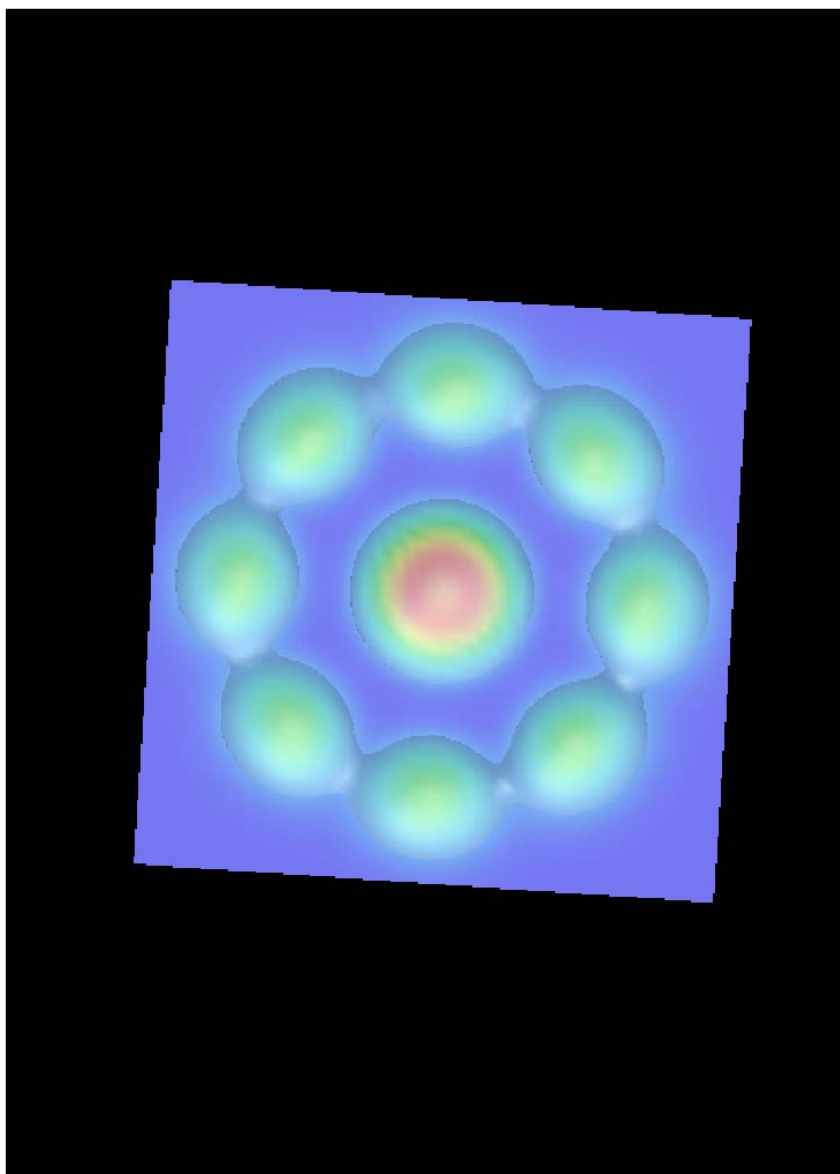
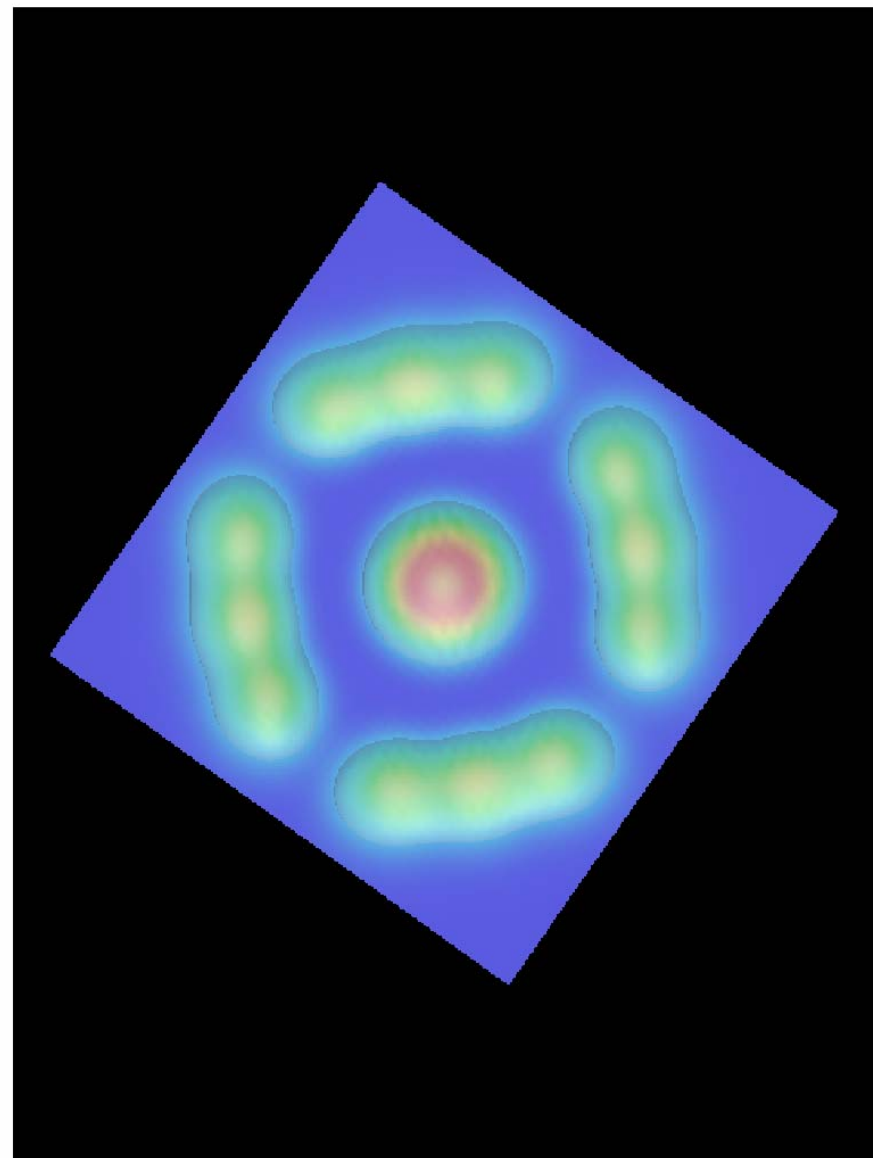
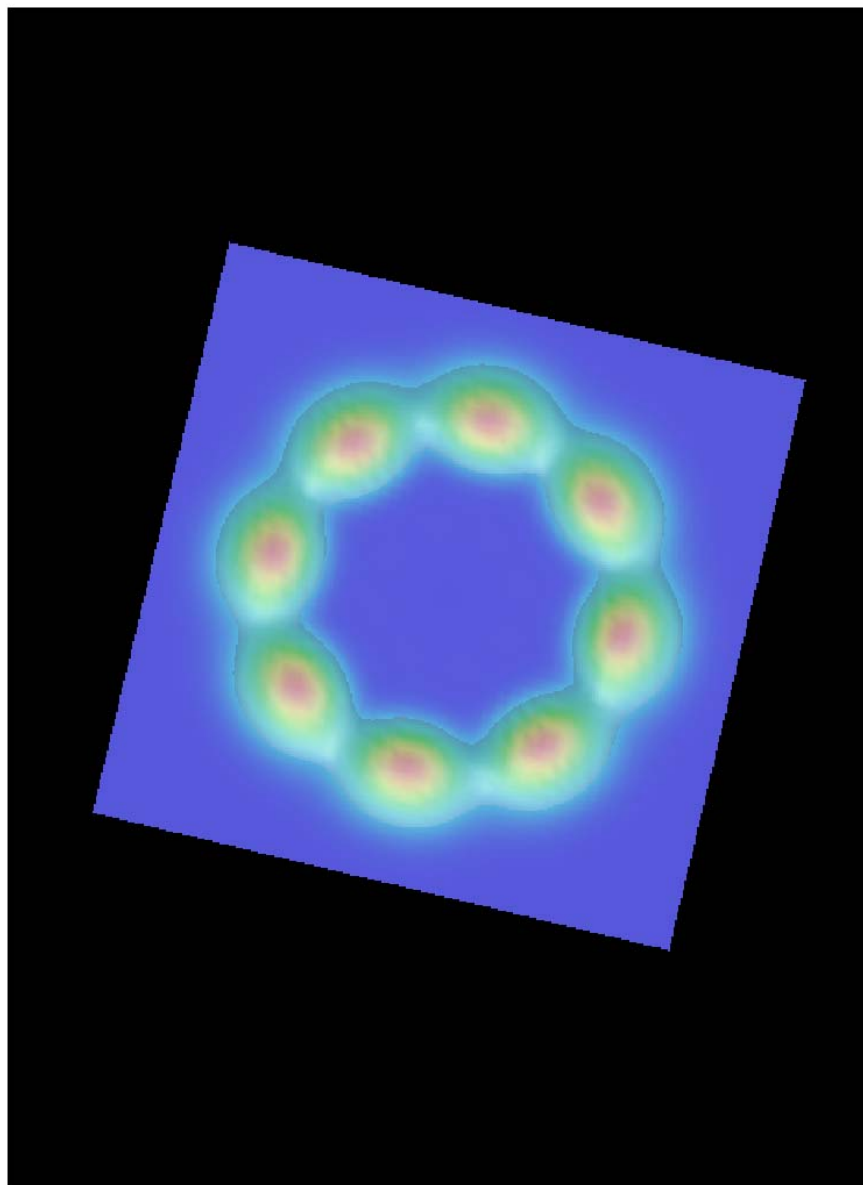
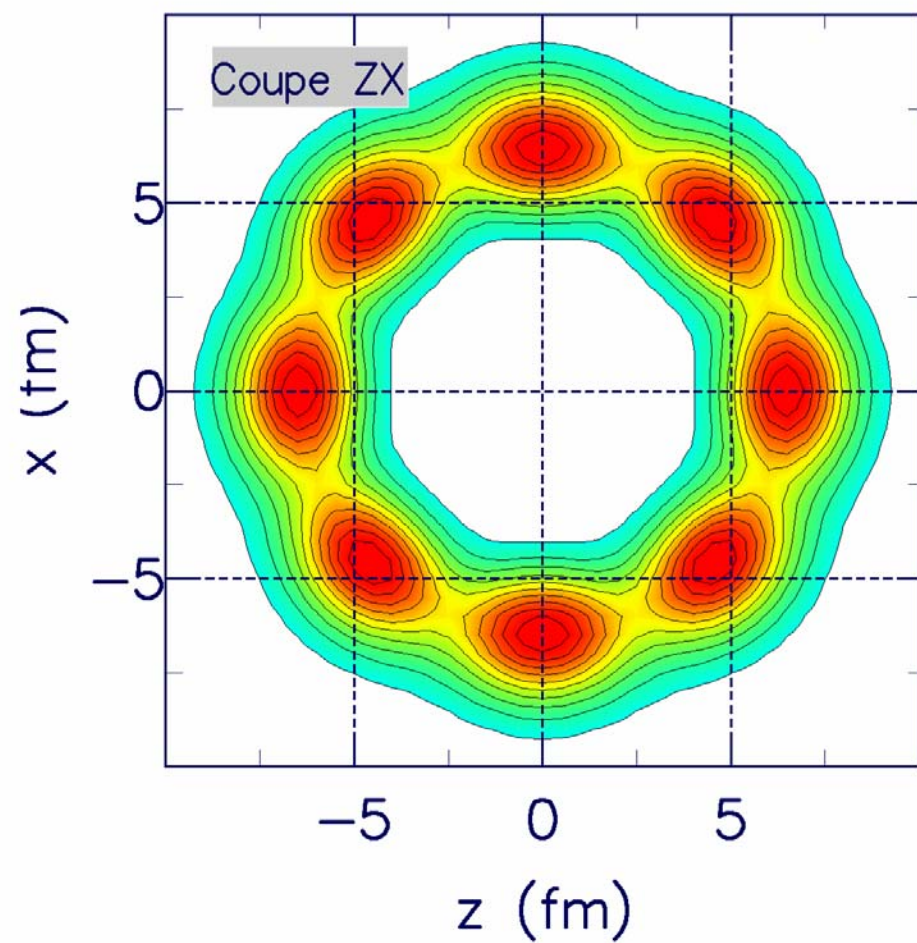


Figure 5.27: The α - α - α correlation function is shown. Resonances from the excited states of ^{12}C are labelled with the first peak seen more clearly in the inner upright panel.

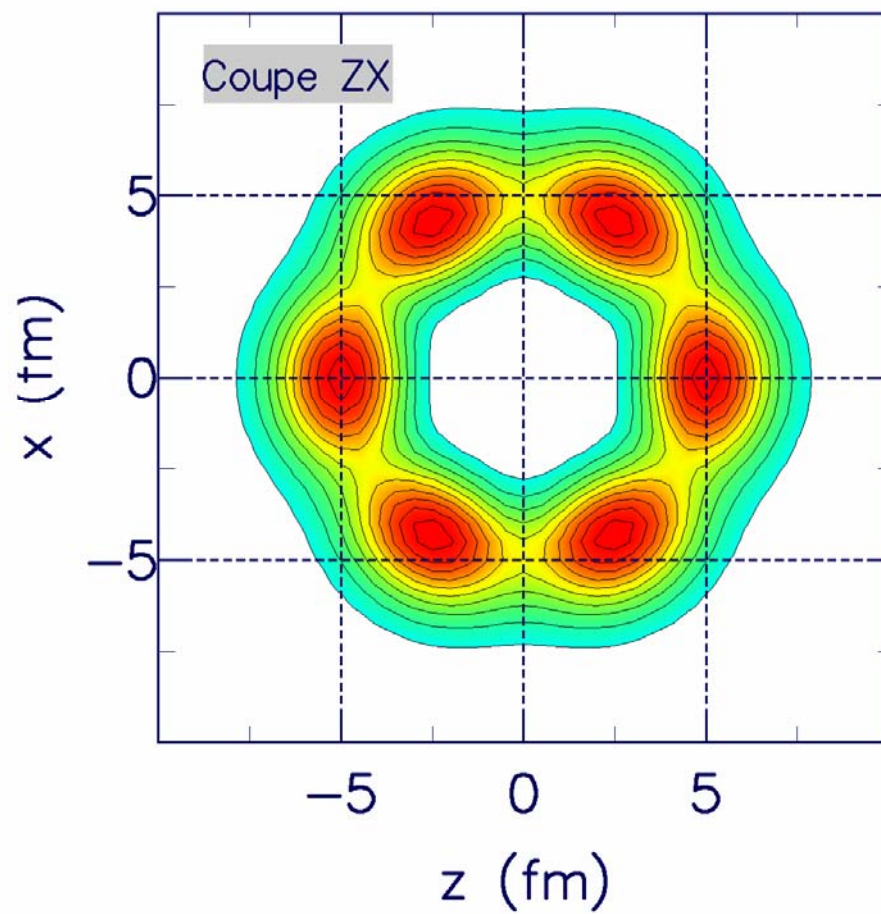




^{32}S



^{24}Mg



^{20}Ne

Coupe ZX

