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Continuum HFB calculations with finite range pairing interactions

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Abstract

A new method of calculating pairing correlations in coordinate space with finite range interactions is presented. In the Hartree–Fock–Bogoliubov (HFB) approach the mean field part is derived from a Skyrme-type force whereas the pairing field is constructed with a Gogny force. An iterative scheme is used for solving the integro-differential HFB equations via the introduction of a local equivalent potential. The method is illustrated on the case of the nucleus ^{18}C. It is shown that the results are insensitive to the cut off value in the quasiparticle spectrum if this value is above 100 MeV.

D.G. Vautherin, M. Veneroni, Phys. Lett. B 25 (1967) 175.

HFB equations in coordinate space

$$hu_1(r) + \int \tilde{h}(r,r')u_2(r')r'^2\,dr' = (E + E_F)u_1(r),$$

$$\int \tilde{h}(r,r')u_1(r')r'^2\,dr' - hu_2(r) = (E - E_F)u_2(r).$$

“Echivalent local potential”

$$\begin{aligned}
&\int \tilde{h}(r,r')u_i(r')r'^2\,dr' \\
&= \frac{1}{u_i(r)}\left(\int \tilde{h}(r,r')u_i(r')r'^2\,dr'\right)u_i(r) \\
&\equiv U_i(r)u_i(r), \quad i = 1, 2.
\end{aligned}$$

Linear interpolation procedure

$$\begin{aligned}
&U_i^{(n+1)}(\epsilon_{n+1}, r) \\
&\quad = R_{\epsilon_{n+1}}\left\{\frac{1}{u_i^{(n)}(r)}\int r'^2\,dr'\,\tilde{h}(r,r')u_i(r')^{(n)}\,dr'\right\},
\end{aligned}$$

Effective Nuclear Forces in a Quark-Meson Couling Model

Outline

- *Basic properties of the quark-meson coupling model*
- *Energy functionals based on QMC model*
- *Predictions for finite nuclei and infinite nuclear matter*

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Physical origin of density dependent forces of Skyrme type within the quark meson coupling model

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Nuclear many-body problem

Calculations based on bare NN force

- the need of effective 3-body forces !?

Models based on effective NN forces

Non-relativistic mean field models

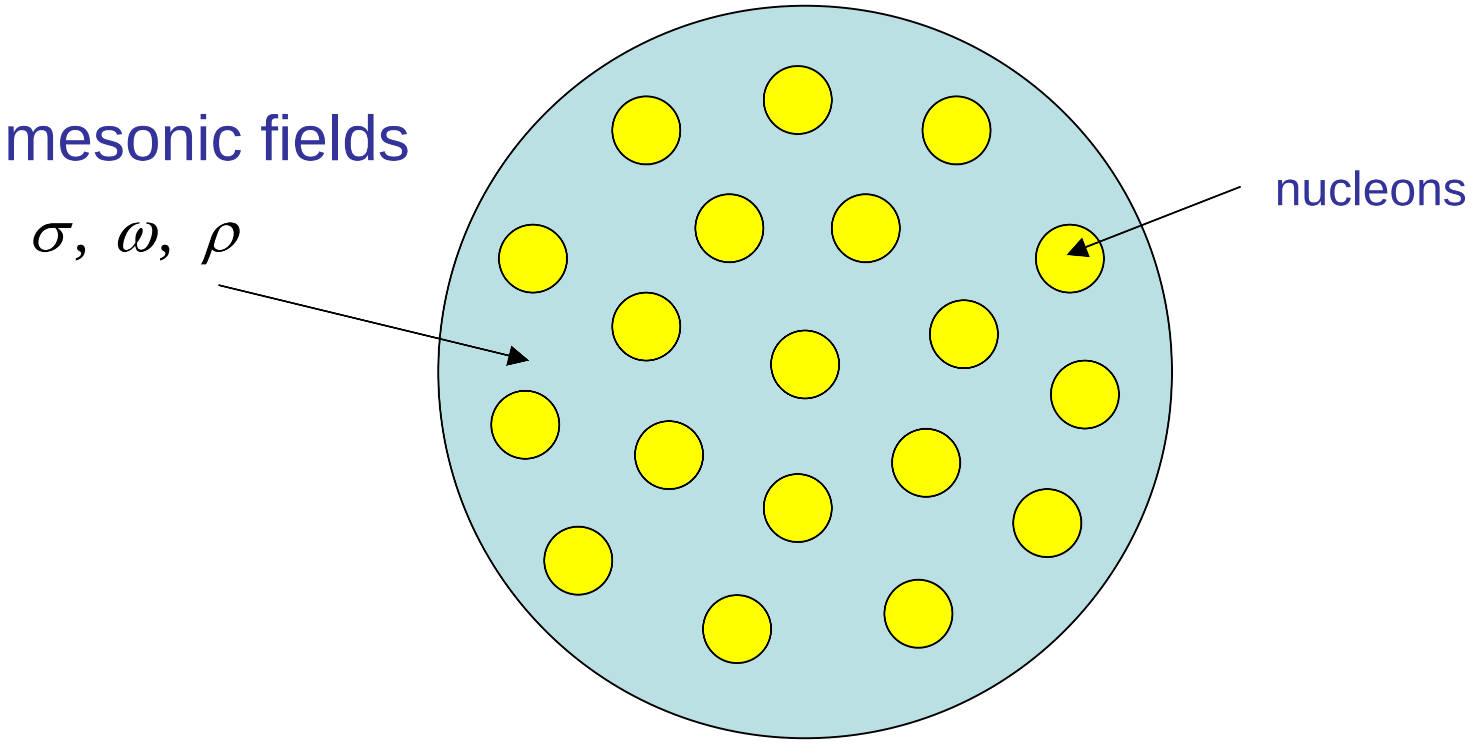
density-dependent two-body forces of Gogny or Skyrme type

Relativistic mean field models

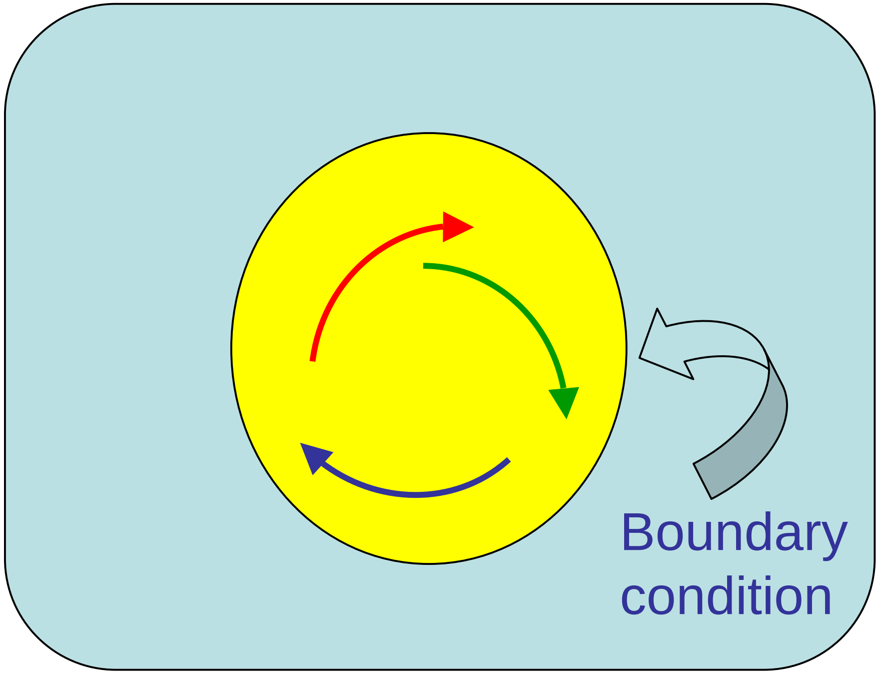
point-like *nucleons interacting by meson exchange*

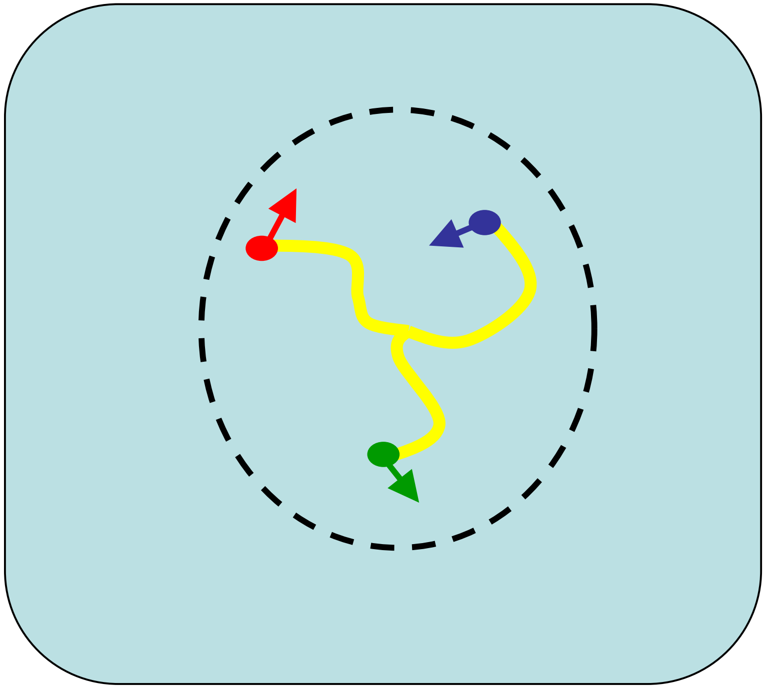
Question: what are the effects of nucleon structure ?

Nuclei with finite size nucleons



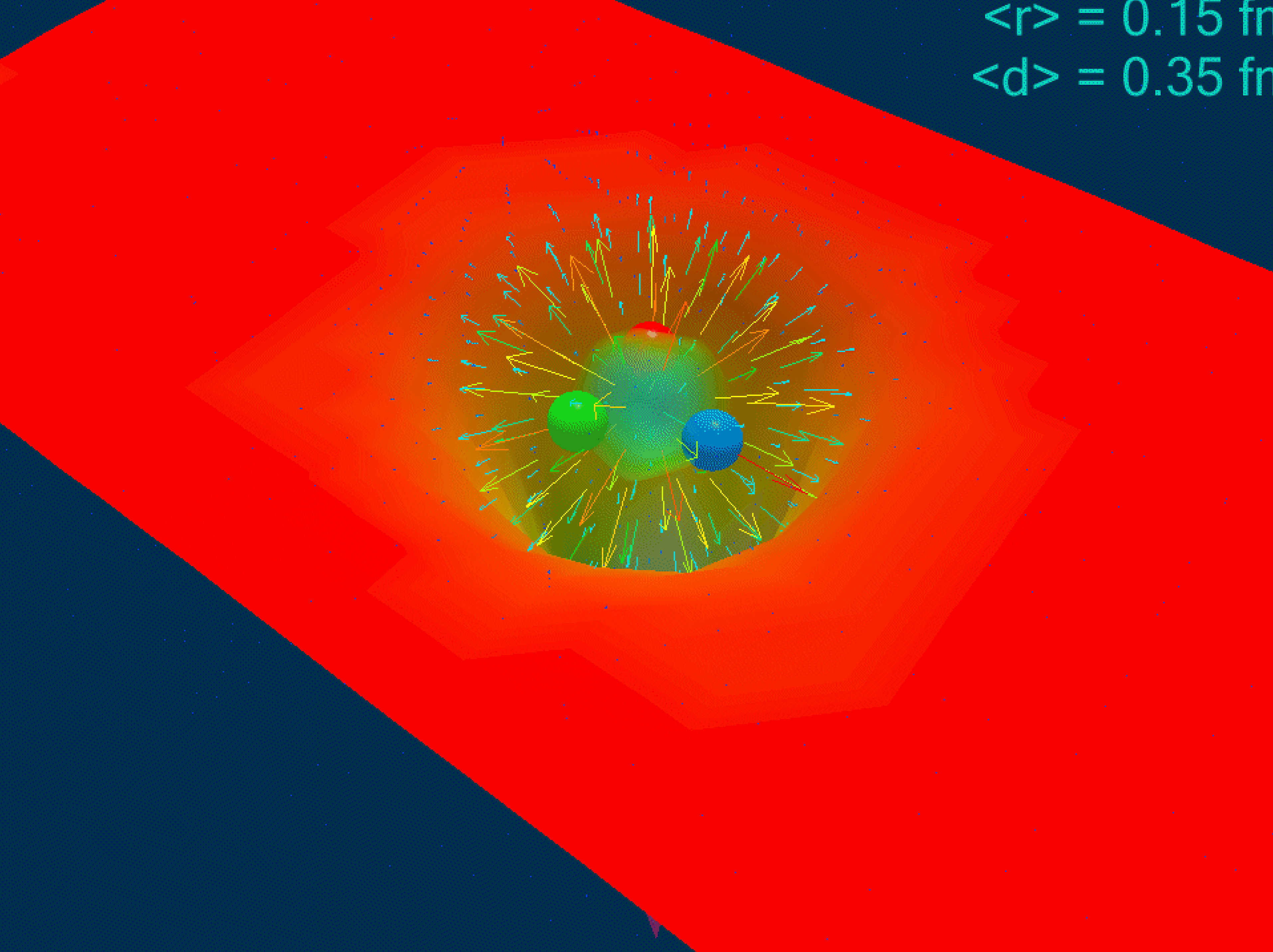
- ***non-overlaping*** **bags of 3 quarks**
- **the quarks interact *locally* with meson fields**





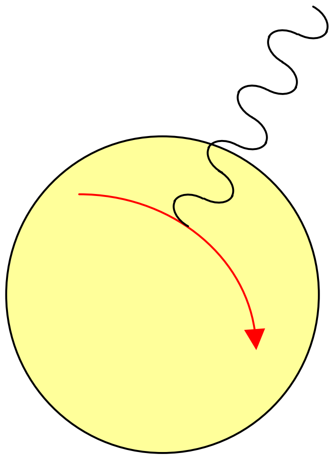
This is just a schematic representation of this

In the true picture, the confined quarks are floating in the non perturbative vacuum. They are coupled to every of its fluctuations.


<r> = 0.15 fm
<d> = 0.35 fm

A static bag in meson field:

$$\begin{cases} -i\gamma^0\vec{\gamma}.\vec{\nabla}\psi + (m_q - g_\sigma^q\sigma)\psi = \dfrac{\Omega(\sigma)}{R_B}\psi \quad (r < R_B) \\ (1+i\vec{\gamma}.\hat{r})\psi(R_B) = 0 \end{cases}$$



$$M(\sigma) = \frac{3\Omega(\sigma)}{R_B} + BV - \frac{Z}{R_B}$$

$$\psi_q(t,r) = e^{-i\frac{\Omega}{R_B}t}\psi(r)$$

$$E = M(\sigma) + g_\omega \omega$$

Numerical study $\Rightarrow$

$$M(\sigma) \approx M(0) - g_\sigma\sigma + \frac{d}{2}(g_\sigma\sigma)^2, \quad d = 0.22R_B$$

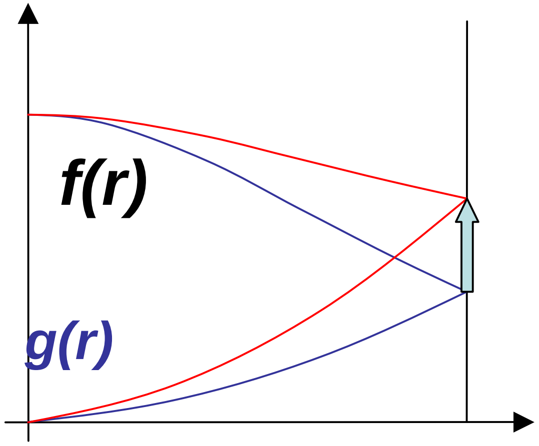
$$g_\sigma \equiv 3g_\sigma^q \int_B d\vec{r}\,\overline{\psi}\psi\Big|_{\sigma=0}$$

Saturation mechanism

$$\sigma(\rho) = \rho(3g_\sigma^q)\int_{Bag} d\vec{r}\,\overline{\psi}\psi\Big|_\sigma$$

$$\omega(\rho) = \rho(3g_\omega^q)\int_{Bag} d\vec{r}\,\psi^+\psi\Big|_\sigma = \rho g_\omega$$

$$\psi(r) = \begin{pmatrix} f(r) \\ i\vec{\sigma}.\hat{r}\,g(r) \end{pmatrix}\chi_{1/2}$$



$$\int_B d\vec{r}\,\overline{\psi}\psi\Big|_\sigma = \int_B d\vec{r}\,(f^2 - g^2)$$

Saturation mechanism due to the change of the quark structure

Total energy for static bags and meson fields

$$E_{tot} = A\left[M(\sigma) + g_\omega \omega\right] + E_{mesons}$$

$$\begin{aligned} E_{mesons} &= \frac{1}{2}\int d\vec{r}\left[(\nabla\sigma)^2 + m_\sigma^2\sigma^2 - (\nabla\omega)^2 - m_\omega^2\omega^2\right] \\ &= \frac{1}{2}V_\infty\left(m_\sigma^2\sigma^2 - m_\omega^2\omega^2\right), \qquad (V_\infty = A/\rho) \end{aligned}$$

$$\frac{\delta E_{tot}}{\delta\sigma} = \frac{\delta E_{tot}}{\delta\omega} = 0$$

Compresibility : K=220 MeV

Including the motion of the nucleonic bags

$$E_{tot} = A\left[M(\sigma) + g_{\omega}\omega\right] + E_{mesons}$$

$$E_{tot} = A\left[\frac{\int_0^{k_F} d\vec{p}\sqrt{p^2 + M^2(\sigma)}}{\int_0^{k_F} d\vec{p}} + g_{\omega}\omega\right] + E_{mesons}$$

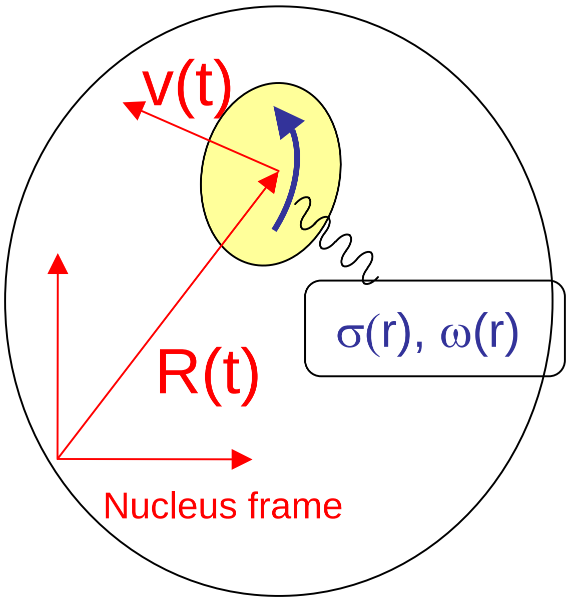
Framework : Born- Oppenheimer Approximation

Nucleon velocity<<Quark velocity

The characteristic time it takes to the quark to adjust its motion is small enough that, during this time, the nucleon motion is locally inertial

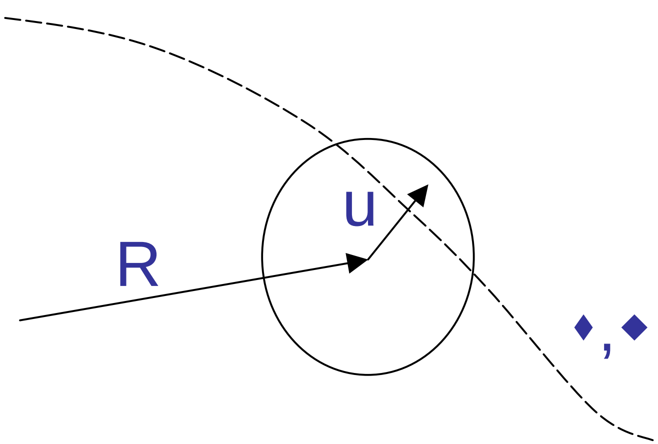
(P.A.M Guichon et al, NPA601,349)

Instantaneous rest frame (IRF)



$$(th\ \xi = \mathrm{v} = \dot{R})$$

Nucleus	IRF
σ	σ
$\omega^0 \equiv \omega$	$\omega\ ch\ \xi$
$\vec{\omega} = 0$	$-\ \omega\ \hat{\mathrm{v}}\ sh\ \xi$



$$\sigma(\vec{R}+\vec{u}) \approx \sigma(\vec{R}) + \vec{u}.\vec{\nabla}\sigma\Big|_R$$

Constant in the bag volume: Treated exactly

Perturbation

Constant part of the fields

$$\begin{cases} E = \sqrt{P^2 + M^2(\sigma)} + g_\omega \omega(\vec{R}) \\ \vec{P} = M(\sigma)\, ch\, \xi\, \hat{v} \end{cases}$$

$$M(\sigma) = M - g_\sigma \sigma + \frac{d}{2}\left(g_\sigma \sigma\right)^2$$

Non-uniform field and Thomas precession

$$V_{SO} = \frac{1}{2M(\sigma)} \vec{\nabla}\left[(2\mu_S - 1) g_\omega \omega - M(\sigma)\right] \mathrm{LS}$$

"magnetic"

Thomas precession

Add the effect of the rho meson $(\,field\ \ b^\alpha, \alpha = 1,2,3)$

$$g_\omega \omega \rightarrow g_\omega \omega + g_\rho \frac{\vec{\tau}}{2}.\vec{b}$$

$$(2\mu_S - 1) g_\omega \omega \rightarrow (2\mu_S - 1) g_\omega \omega + (2\mu_V - 1) g_\rho \frac{\vec{\tau}}{2}.\vec{b}$$

Total energy of the system

$$E = \sum_i \left(\sqrt{P_i^2 + M_{\text{eff}}(\sigma)^2} + g_\omega \omega + V_{\text{so}} \right) + E_{\text{mesons}}$$

$$M_{\text{eff}}(\sigma) = M - g_\sigma \sigma + \frac{d}{2}(g_\sigma \sigma)^2$$

$$E_{\text{mesons}} = \frac{1}{2}\int d\vec{r}\,\left[(\nabla\sigma)^2 + m_\sigma^2 \sigma^2\right] - \frac{1}{2}\int d\vec{r}\,\left[(\nabla\omega)^2 + m_\omega^2 \omega^2\right]$$

Hamiltonian :

$$H(R_i, P_i) = E(R_i, P_i, \sigma \to \sigma_{\text{sol}}, \omega \to \omega_{\text{sol}})$$

$$\frac{\delta E}{\delta \sigma} = \frac{\delta E}{\delta \omega} = 0$$

Quantisation ?

$\sigma(r) \equiv \sigma(r, R_1, R_2, ...)$ corresponds to a many body operator !!!

The solution of the field equations

1) Mean field: replace all operators by their ground state expectation value

   drawback: loose correlations

2) Expand the operators around their ground state expectation value and solve the equations for the fluctuations

Equation for sigma field

$$\boxed{-\nabla^2\sigma + m_\sigma^2\sigma = -\frac{\partial}{\partial\sigma(r)}K(\sigma)}$$

$$K(\sigma) = \sum_i \delta(r - R_i)\sqrt{P_i^2 + M[\sigma(r)]}$$

Expand: $\sigma = \bar{\sigma} + \delta\sigma \qquad (\bar{\sigma} \equiv <\sigma>)$

$$(-\nabla^2 + m_\sigma^2)(\bar{\sigma} + \delta\sigma) = -\frac{\partial K}{\partial\sigma}(\bar{\sigma}) - \delta\sigma\frac{\partial^2 K}{\partial\sigma^2}(\bar{\sigma}) - \ldots$$

$$\frac{\partial K}{\partial\sigma}(\bar{\sigma}) = <\frac{\partial K}{\partial\sigma}(\bar{\sigma})> + \delta\left[\frac{\partial K}{\partial\sigma}(\bar{\sigma})\right]$$

$$\frac{\partial^2 K}{\partial\sigma^2}(\bar{\sigma}) = <\frac{\partial^2 K}{\partial\sigma^2}(\bar{\sigma})> + \delta\left[\frac{\partial^2 K}{\partial\sigma^2}(\bar{\sigma})\right]$$

Solve the equation order by order:

$$(-\nabla^2 + m_\sigma^2)\bar{\sigma} = -< \frac{\partial K}{\partial \sigma}(\bar{\sigma}) > \qquad \text{(mean field)}$$

$$(-\nabla^2 + m_\sigma^2 + < \frac{\partial^2 K}{\partial \sigma^2}(\bar{\sigma}) >)\delta\sigma = -\frac{\partial K}{\partial \sigma}(\bar{\sigma}) - < \frac{\partial K}{\partial \sigma}(\bar{\sigma}) > \qquad \text{(fluctuation)}$$

Plug this into the energy:

$$H = E(\sigma \to \sigma_{solution})$$

$$= \int d^3r\, K(\bar{\sigma}) - \frac{1}{2}\bar{\sigma} < \frac{\partial K}{\partial \sigma}(\bar{\sigma}) > + \frac{1}{2}\delta\sigma\left(\frac{\partial K}{\partial \sigma}(\bar{\sigma}) - < \frac{\partial K}{\partial \sigma}(\bar{\sigma}) > \right)$$

(+ terms of order ♎^{3})

the quantum expression for *K* (and derivatives)

$$K(\overline{\sigma}) = \sum_i \delta(r - R_i)\sqrt{P_i^2 + M[\overline{\sigma}(r)]^2}$$ is a one body operator

$$K(\overline{\sigma}) = \sum_{\alpha\beta} < \alpha \left| K(\overline{\sigma}) \right| \beta > a_\alpha^+ a_\beta$$

Natural choice in momentum basis:

$$K[\overline{\sigma}(\vec{r})] = \frac{1}{2V} \sum_{\vec{k},\vec{k}'} e^{i(\vec{k}-\vec{k}').\vec{r}} \left[\sqrt{k^2 + M[\overline{\sigma}(\vec{r})]^2} + k \leftrightarrow k' \right] a_{\vec{k}}^+ a_{\vec{k}'}$$

Finite nuclei case

- Non-relativistic expansion
- Expand in power of g_σ up to second order
- HF approximation

Energy functional for finite nuclei

$$\langle H(\vec{r}\,)\rangle = \rho M + \frac{\tau}{2M} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\rm eff} + \mathcal{H}_{\rm fin} + \mathcal{H}_{\rm so},$$

$$\mathcal{H}_0 + \mathcal{H}_3 = \rho^2\left[\frac{-3G_\rho}{32} + \frac{G_\sigma}{8(1+d\rho G_\sigma)^3} - \frac{G_\sigma}{2(1+d\rho G_\sigma)} + \frac{3G_\omega}{8}\right]$$
$$+ (\rho_n - \rho_p)^2\left[\frac{5G_\rho}{32} + \frac{G_\sigma}{8(1+d\rho G_\sigma)^3} - \frac{G_\omega}{8}\right],$$

$$\mathcal{H}_{\rm eff} = \left[\left(\frac{G_\rho}{8{m_\rho}^2} - \frac{G_\sigma}{2{m_\sigma}^2} + \frac{G_\omega}{2{m_\omega}^2} + \frac{G_\sigma}{4{M_N}^2}\right)\rho_n + \left(\frac{G_\rho}{4m_\rho^2} + \frac{G_\sigma}{2M_N^2}\right)\rho_p\right]\tau_n$$
$$+ p \leftrightarrow n,$$

$$\mathcal{H}_{\rm fin} = \left[\left(\frac{3G_\rho}{32{m_\rho}^2} - \frac{3G_\sigma}{8{m_\sigma}^2} + \frac{3G_\omega}{8{m_\omega}^2} - \frac{G_\sigma}{8{M_N}^2}\right)\rho_n\right.$$
$$\left. + \left(\frac{-3G_\rho}{16{m_\rho}^2} - \frac{G_\sigma}{2{m_\sigma}^2} + \frac{G_\omega}{2{m_\omega}^2} - \frac{G_\sigma}{4{M_N}^2}\right)\rho_p\right]\nabla^2(\rho_n) + p \leftrightarrow n,$$

Spin-orbit energy density

$$V_{so}^{QMC} = \frac{1}{32M^2}\{\left(8G_\sigma + 8G_\omega(2\mu_s - 1)\right)\vec{J}_p.\vec{\nabla}\rho + \left(4G_\sigma + 4G_\omega(2\mu_s - 1) + 3G_\rho(2\mu_v - 1)\right)\vec{J}_p.\vec{\nabla}\rho_p$$
$$+ p \leftrightarrow n\}$$

$$V_{so}^{Skyrme} = \frac{W_0}{2}\left[\vec{J}_p.\vec{\nabla}\rho + \vec{J}_p.\vec{\nabla}\rho_p + p \leftrightarrow n\right]$$

$$W_p^{Skyrme} = \frac{W_0}{2}[2\nabla\rho_p + \nabla\rho_n]$$

$$W_p^{QMC} \propto [2.8\nabla\rho_p + \nabla\rho_n]$$

$$W_p^{RMF} \propto [\nabla\rho_p + \nabla\rho_n]$$

Parameters

$G_\sigma, G_\omega, G_\rho$ are fixed by the saturation point and the symmetry energy.

G_σ	G_ω	G_ρ
12.3 fm^2	8.9 fm^2	7.7 fm^2

Compressibility:~345MeV

Binding energies given by HF-QMC

	E_B (MeV, exp)	E_B (MeV, QMC)	r_C (fm, exp)	r_C (fm, QMC)
^{16}O	7.976	7.618	2.73	2.702
^{40}Ca	8.551	8.213	3.485	3.415
^{48}Ca	8.666	8.343	3.484	3.468
^{208}Pb	7.867	7.515	5.5	5.42

Spin-orbit splitting given by HF-QMC

	Neutrons (exp)	Neutrons (QMC)	Protons (exp)	Protons (QMC)
^{16}O, $1p_{1/2}$–$1p_{3/2}$	6.10	6.01	6.3	5.9
^{40}Ca, $1d_{3/2}$–$1d_{5/2}$	6.15	6.41	6.00	6.24
^{48}Ca, $1d_{3/2}$–$1d_{5/2}$	6.05 (Sly4)	5.64	6.06 (Sly4)	5.59
^{208}Pb, $2d_{3/2}$–$2d_{5/2}$	2.15 (Sly4)	2.04	1.87 (Sly4)	1.74

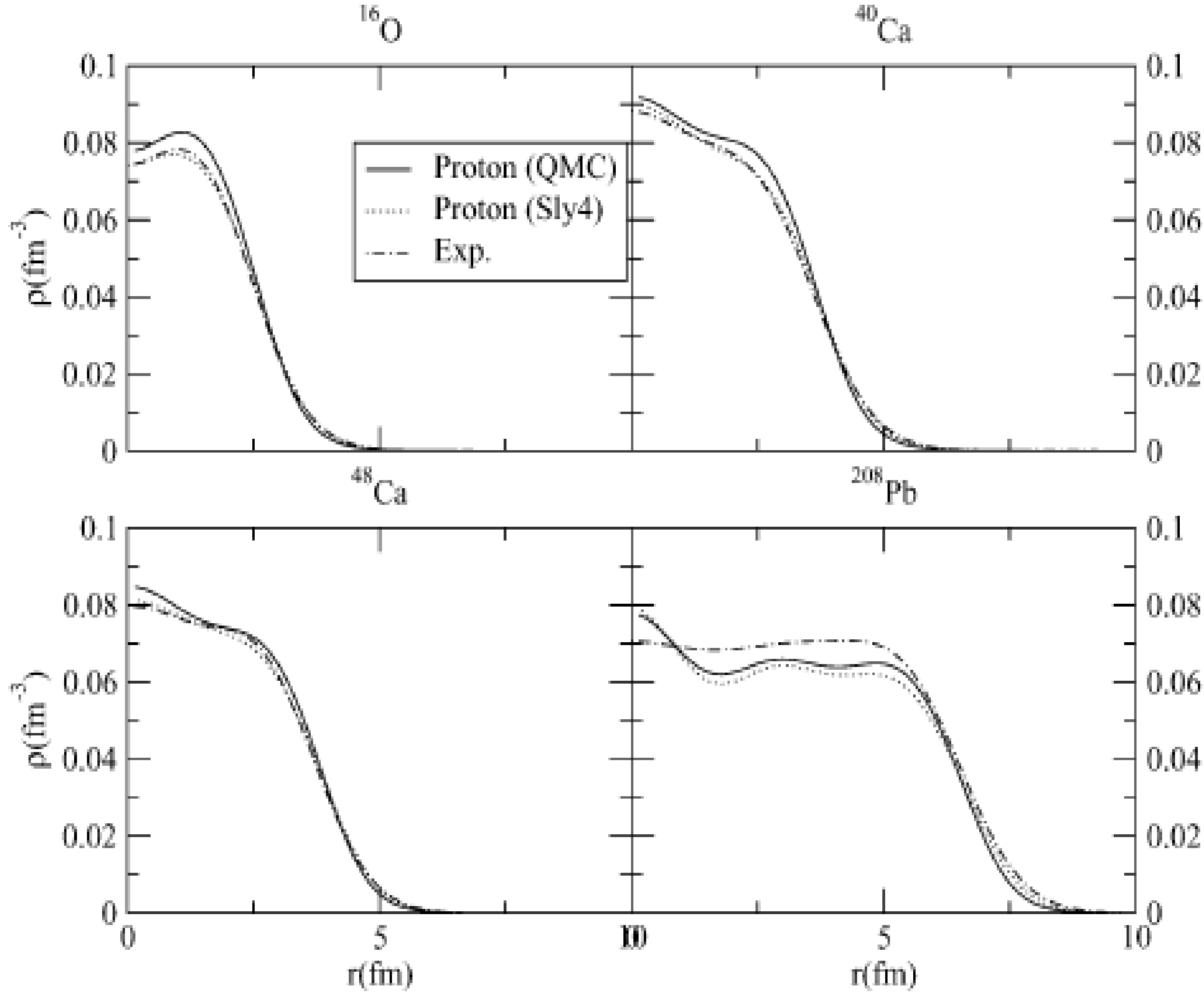


Fig. 1. Proton densities of the QMC model compared with experiment and the prediction of the Skyrme Sly4 force.

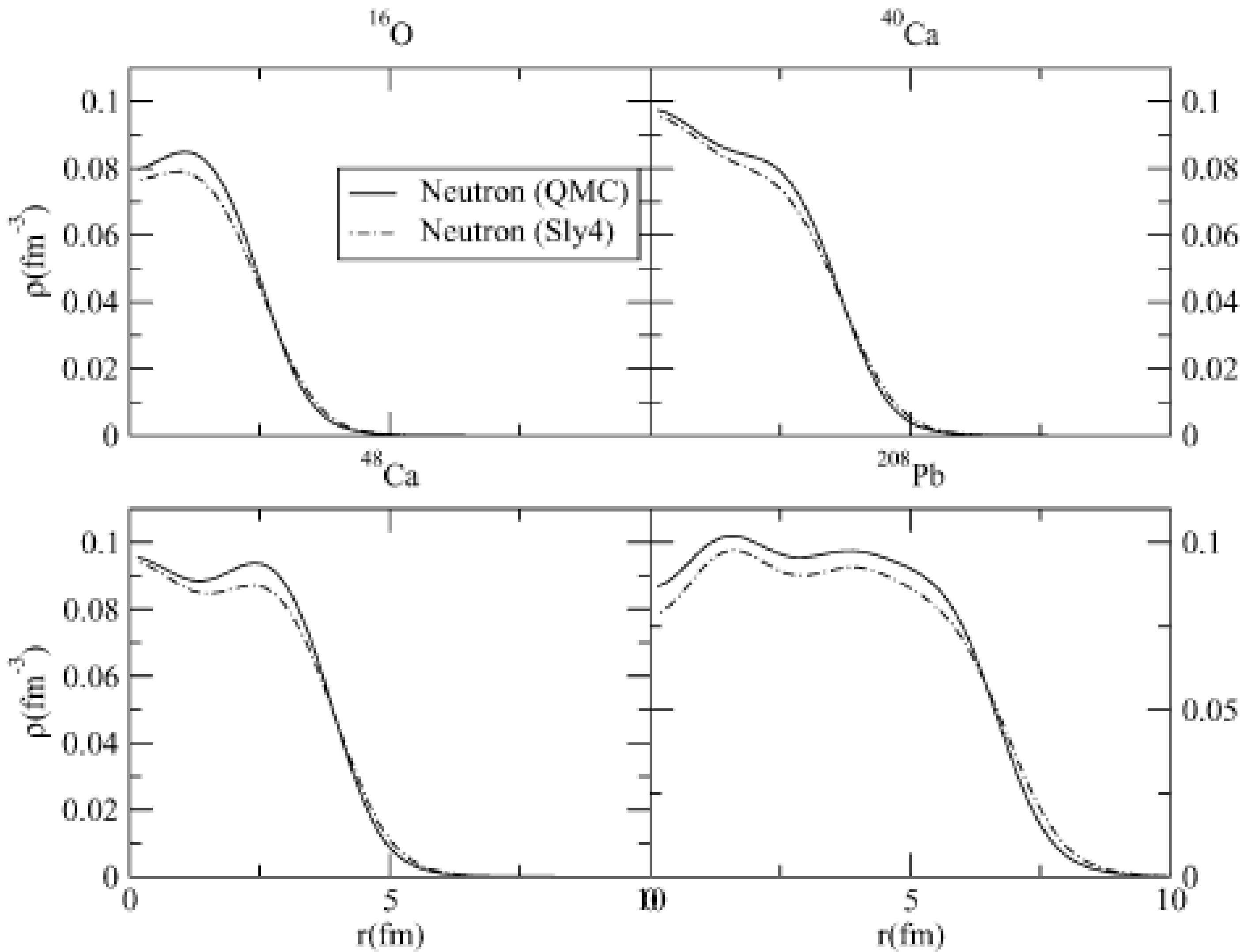


Fig. 2. Neutron densities of the QMC model compared with the prediction of the Skyrme Sly4 force.

Nuclei far from stability

Two-neutron drip line

Ni *isotopes : N=60*

Zr *isotopes: N=82*

Similar predictions as SLy4

Shell quenching

$\underline{S_{2n}\text{ across } N=28 :}$

8 MeV for Z= 32

3 MeV for Z= 14

Symmetric nuclear matter

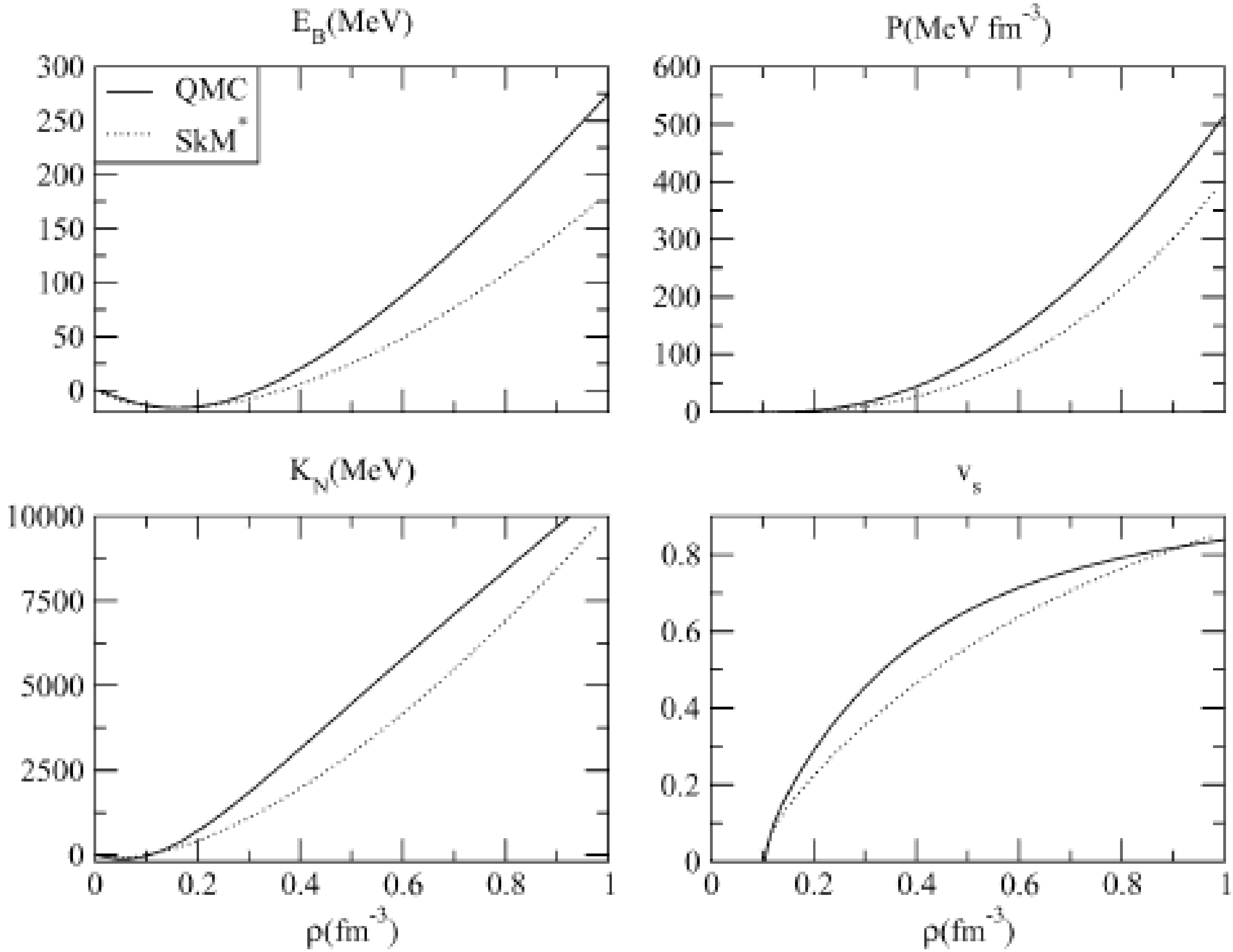


Fig. 3. Comparison of the QMC and SkM* model for symmetric matter.

Neutron matter

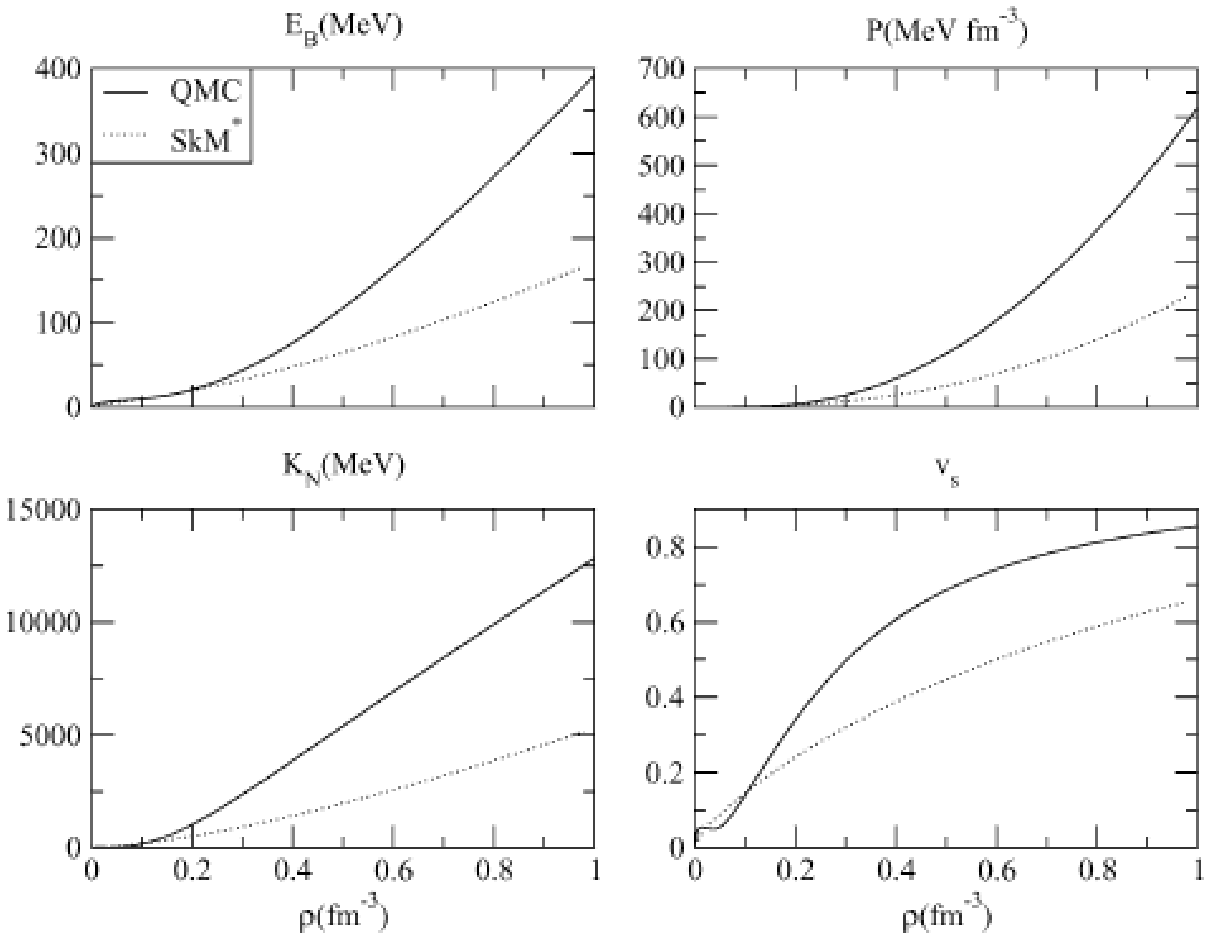


Fig. 4. Comparison of the QMC and SkM* model for neutron matter.

Conclusions

• The QMC energy functional has a similar structure with Skyrme functionals

• Different density and isospin dependence !!