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Theory and simulation of large scale structure formation in the universe

Romain Teyssier

CEA/DSM/DAPNIA/SAp
http://www.projet-horizon.fr

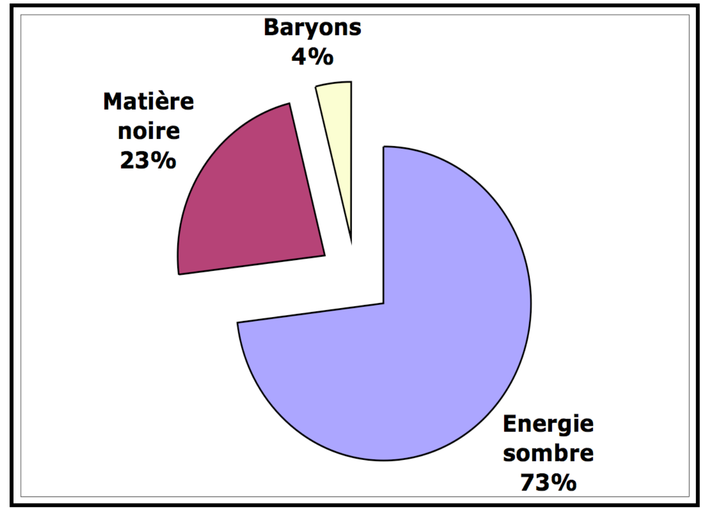
Initial conditions: random (Gaussian) density fluctuations

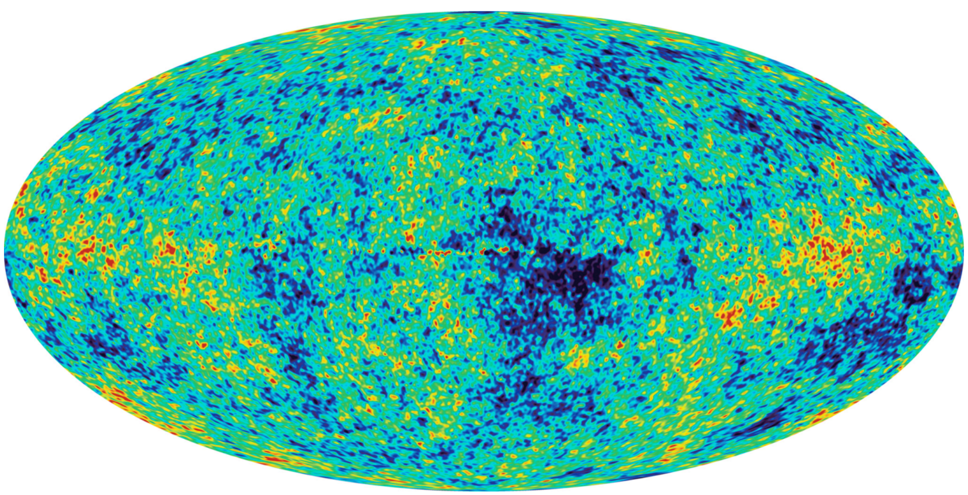
Seeds for structure (galaxy) formation

Fluctuations at quantum scales

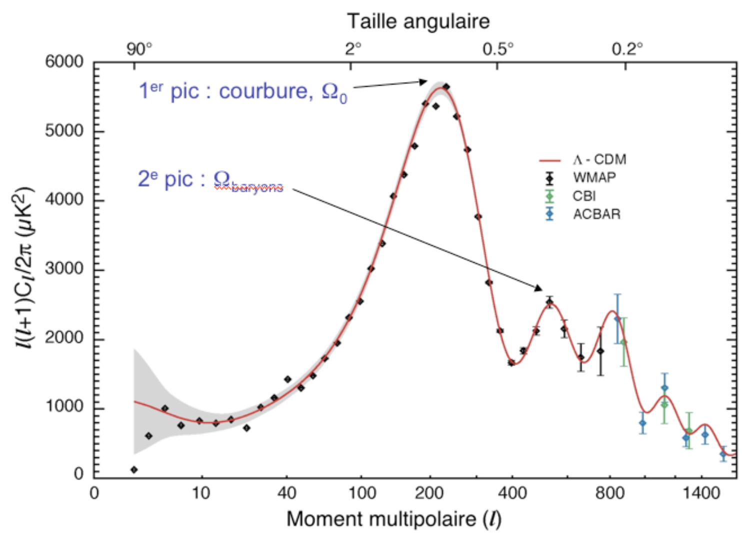
Amplified by « Inflation » up to cosmic scales

Observed on the diffuse microwave background at a 10^{-5} level



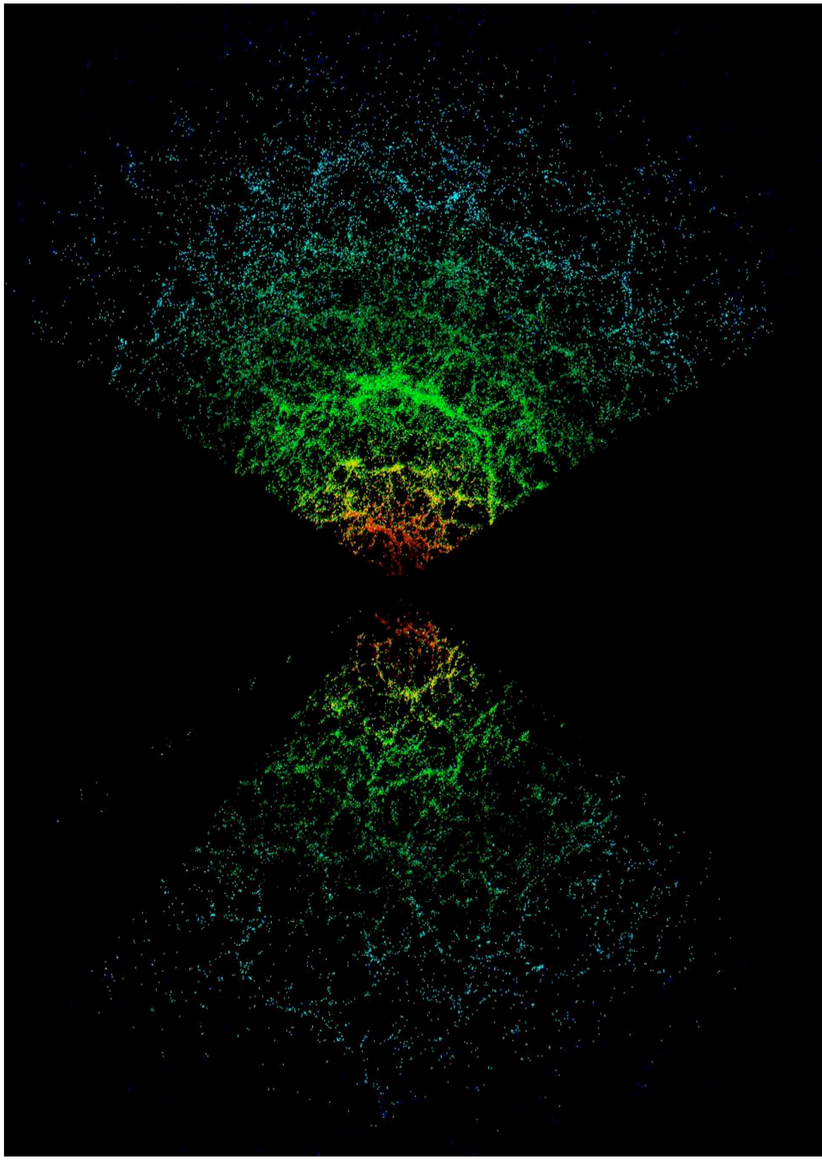


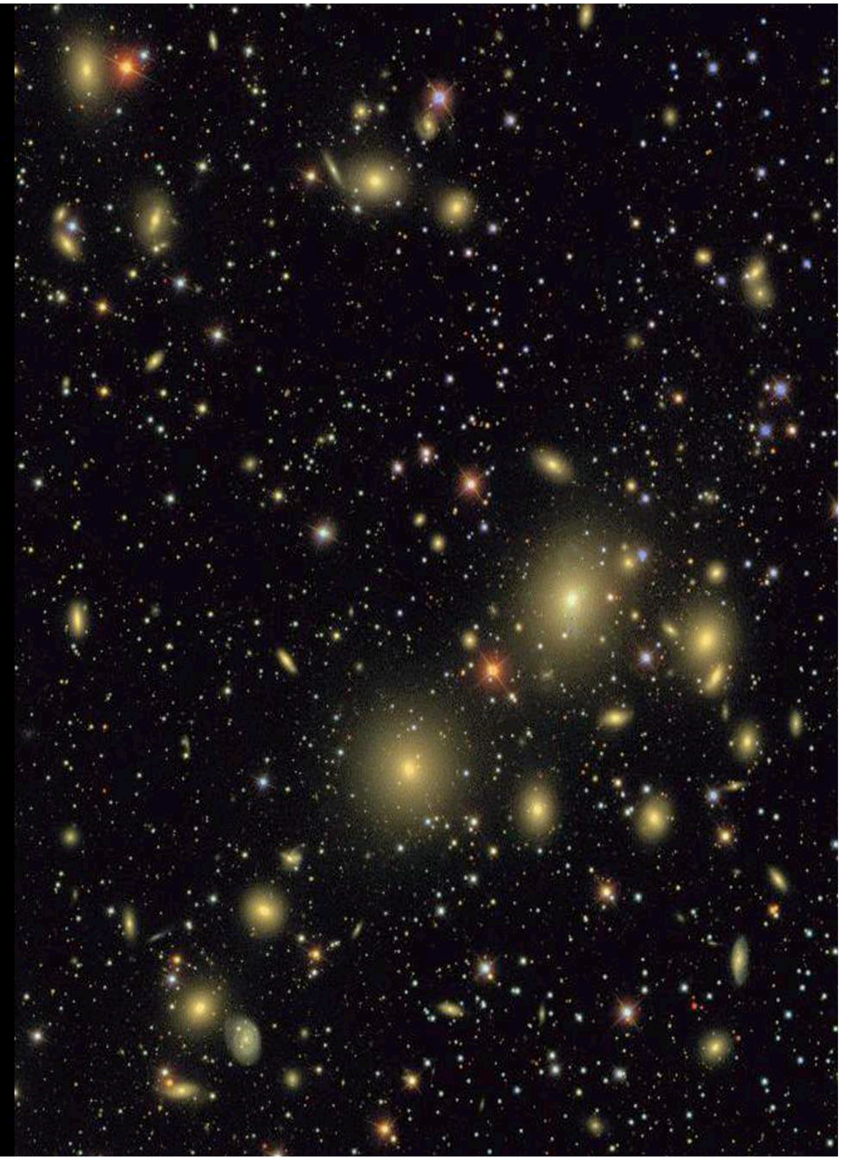
Le satellite WMAP a observé l'univers lorsqu'il avait 300 000 ans



La position du premier pic indique que la densité totale de matière et d'énergie vaut : $\Omega_0 = 1.02 \pm 0.02$ (WMAP 2/2003).

Galaxies are dispatched along the « cosmic web »





Sloan Digital Sky Survey 2003

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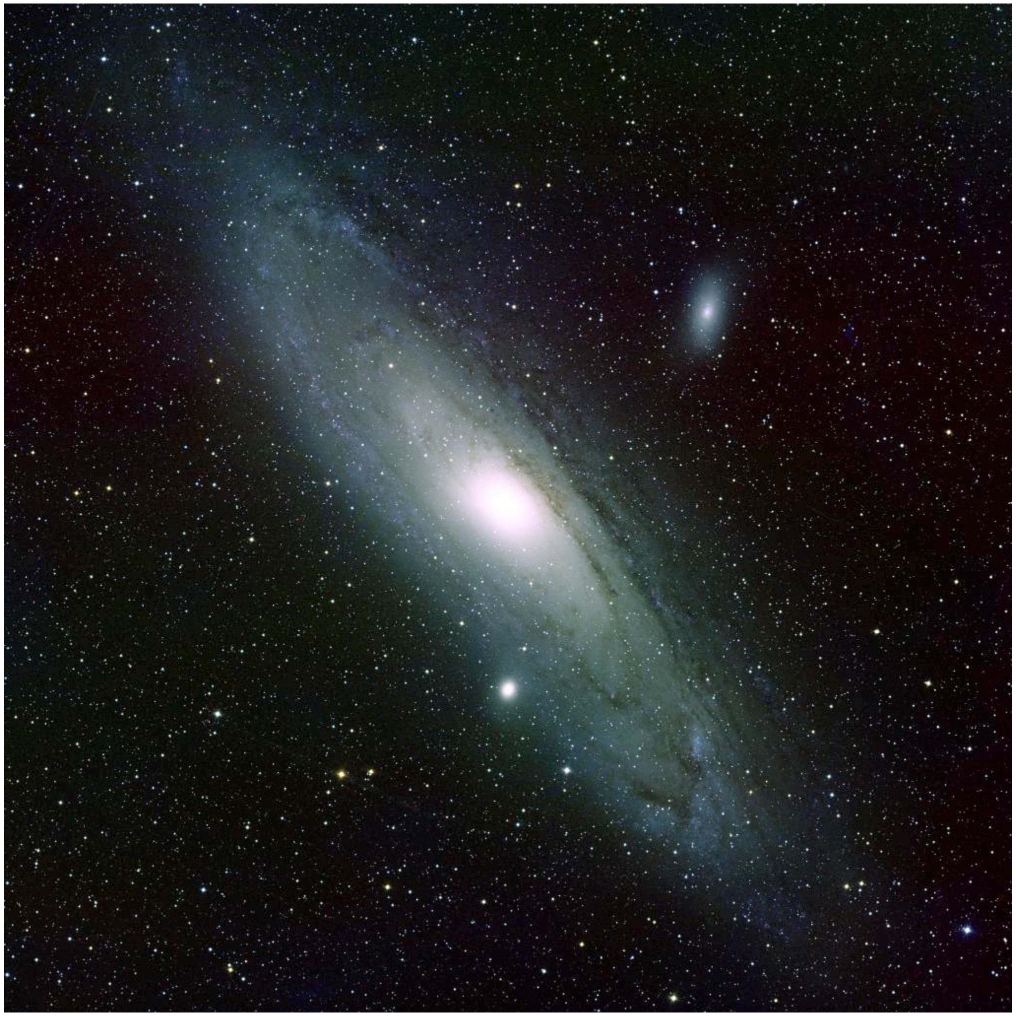
Galaxies have complex internal structure



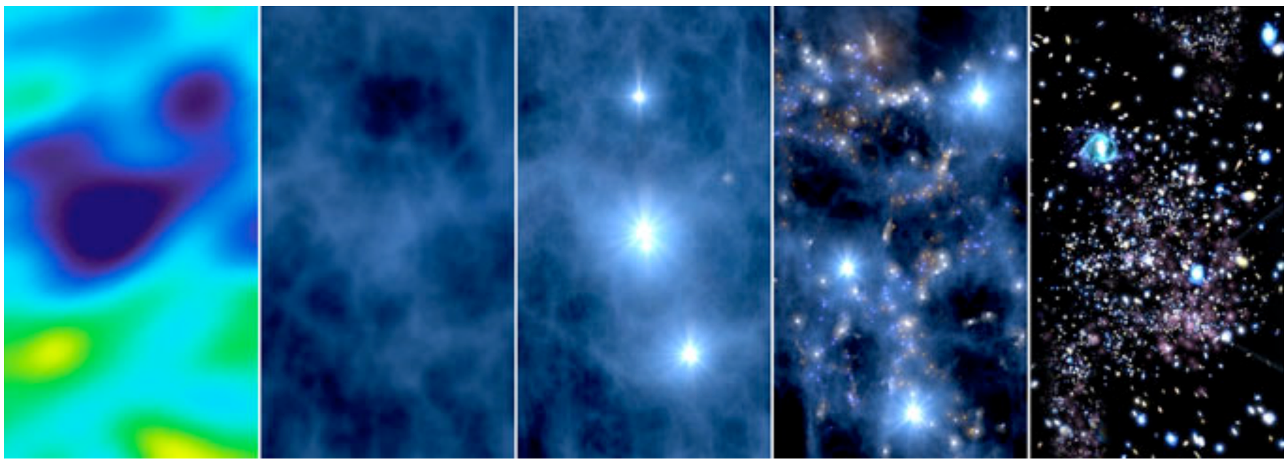
Mathematical model: self-gravitating fluid dynamics

Gravitational instability amplifies seed random fluctuations

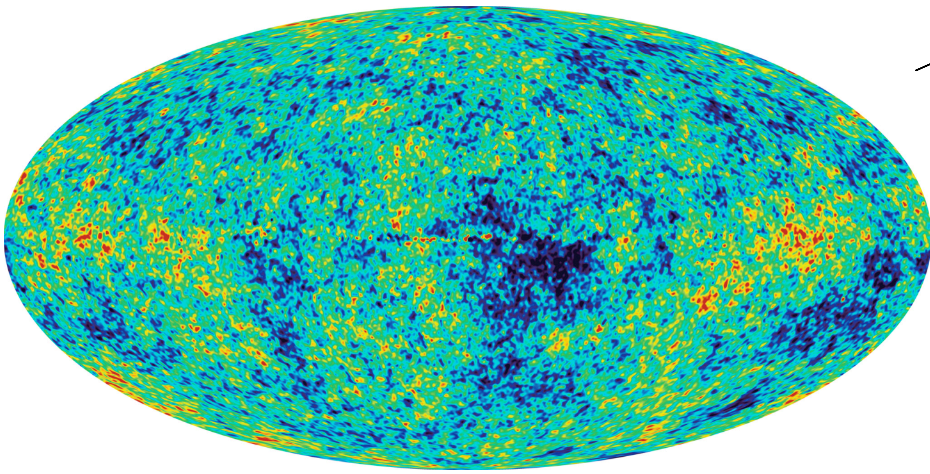
Dark matter collisionless gravitational dynamics at large scale
Gas physics dominates at small scales

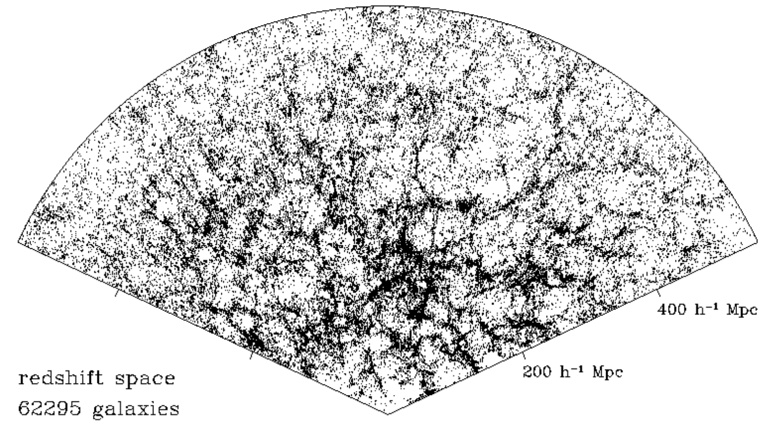
Hierarchical scenario for structure formation:

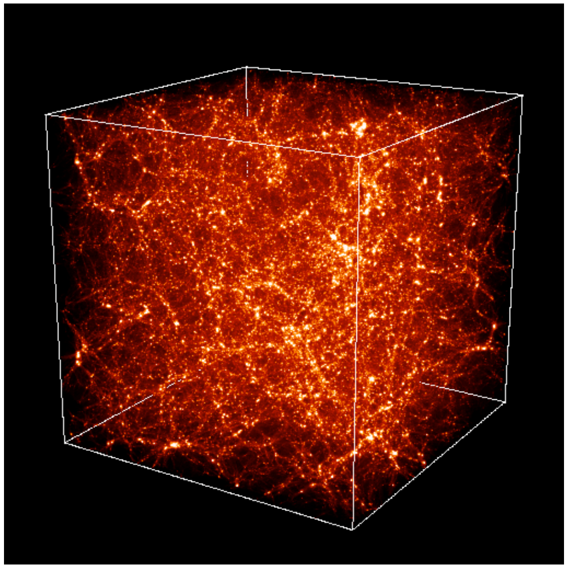
Dark Ages
Dwarf galaxies
Massive galaxies
Galaxy clusters



Origin of structure ?







Gravitational dynamics is a complex, non-linear process

Use of massive N body simulations

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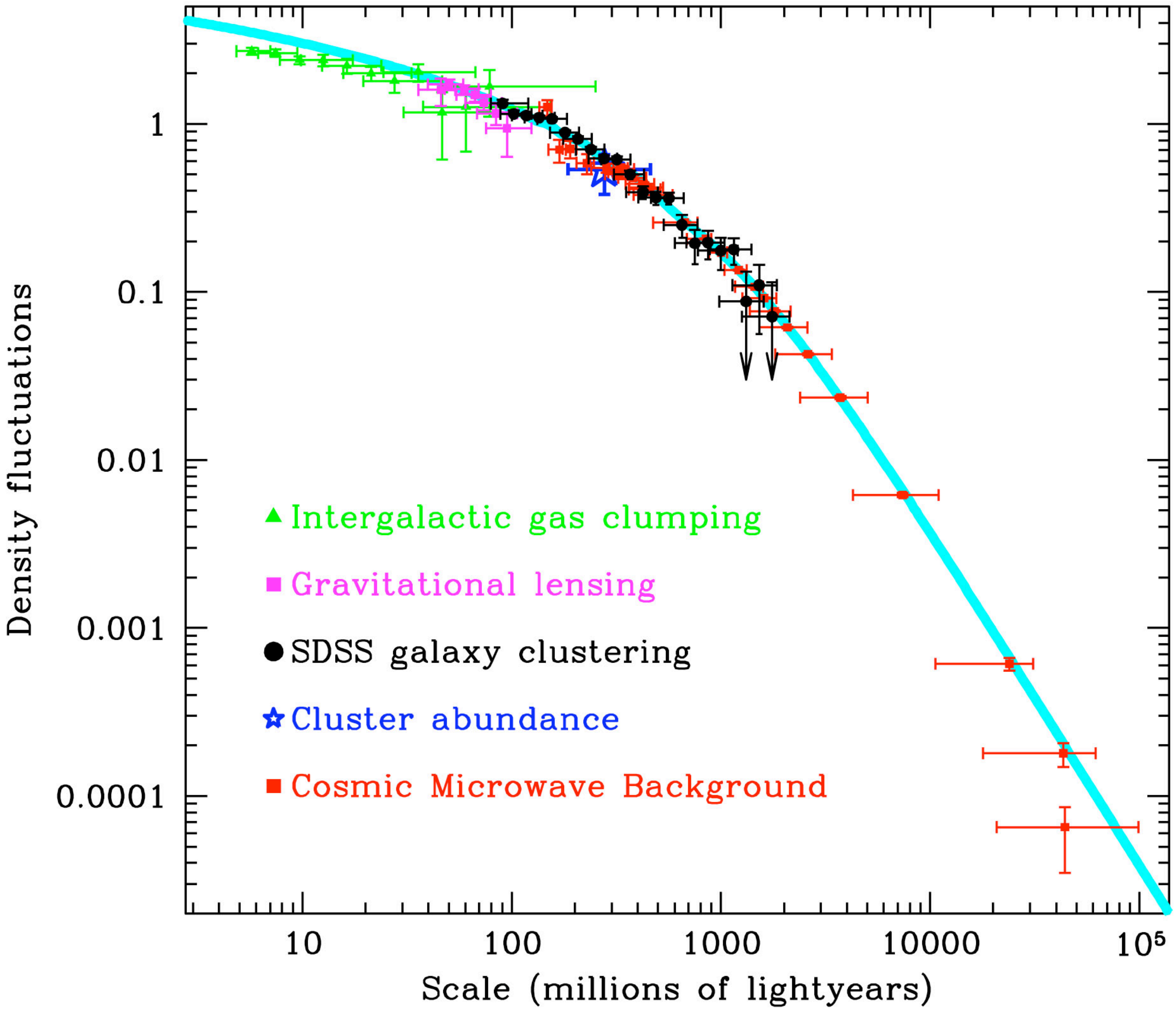
Density distribution using different probes



A cook book for building virtual universes

- Dark matter (and stars) follows the Vlasov-Poisson equations
  $\Rightarrow$ limit of infinite collision time for Boltzmann equation

- Baryons dynamics follow the Euler-Poisson equations
  $\Rightarrow$ limit of infinite collision rate for Boltzmann equation

- Radiative transfer: atomic and molecular cooling, UV heating, ionization fronts, dust absorption $\Rightarrow$ radiative feedback

- Star formation recipe (phenomenological)

- Supernovae driven winds, AGN jets $\Rightarrow$ thermal and kinetic feedback

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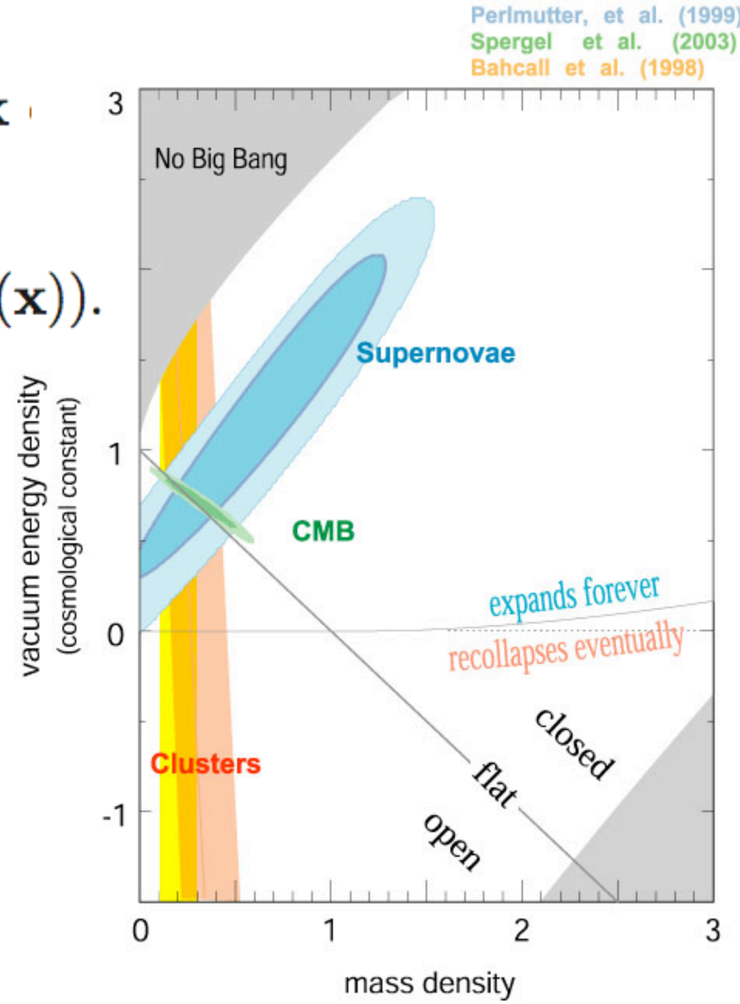
Collisionless N body system in an expanding universe

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho. \qquad H \equiv \frac{\dot{a}}{a}. \qquad \mathbf{r} = a\,\mathbf{x}$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = G\int \mathrm{d}^3\mathbf{r}'\,\frac{\rho(\mathbf{r}')(\mathbf{r}'-\mathbf{r})}{|\mathbf{r}'-\mathbf{r}|^3} \qquad \rho(\mathbf{x}) = \overline{\rho}(1+\delta(\mathbf{x})).$$

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\dot{a}}{a}\mathbf{u} = \mathbf{g} = G\overline{\rho}\,a\int \mathrm{d}^3\mathbf{x}'\,\frac{\delta(\mathbf{x}')(\mathbf{x}'-\mathbf{x})}{|\mathbf{x}'-\mathbf{x}|^3}.$$



$$\mathbf{p} = \mathbf{u}\,m\,a. \qquad f(\mathbf{x},\mathbf{p})\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{p},$$

$$\Delta\Phi(\mathbf{x}) = \frac{4\pi G m}{a}\left(\int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p} - \overline{n}\right),$$

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial}{\partial t}f(\mathbf{x},\mathbf{p},t) + \frac{\mathbf{p}}{ma^2}\frac{\partial}{\partial \mathbf{x}}f(\mathbf{x},\mathbf{p},t) - m\nabla_{\mathbf{x}}.\Phi(\mathbf{x})\frac{\partial}{\partial \mathbf{p}}f(\mathbf{x},\mathbf{p},t) = 0$$

Single stream flow and the linear regime

$$\mathbf{u} = \frac{\int d^3\mathbf{p}\,\mathbf{p}\,f(\mathbf{x},\mathbf{p})}{m\,a\int f\,d^3\mathbf{p}}, \qquad \frac{\int d^3\mathbf{p}\,\mathbf{p}_i\,\mathbf{p}_j\,f(\mathbf{x},\mathbf{p})}{m^2\,a^2\int f\,d^3\mathbf{p}} = \mathbf{u}_i\,\mathbf{u}_j + \sigma_{ij}.$$

$$\begin{cases} \dfrac{\partial\delta}{\partial t} + \dfrac{1}{a}\nabla_{\mathbf{x}}.\left[(1+\delta(\mathbf{x}))\mathbf{u}(\mathbf{x})\right] = 0, \\[2ex] \dfrac{\partial\mathbf{u}_i}{\partial t} + \dfrac{\dot{a}}{a}\mathbf{u}_i + \dfrac{1}{a}(\mathbf{u}_j.\nabla_j)\mathbf{u}_i = -\dfrac{1}{a}\nabla_i\Phi - \dfrac{1}{\rho\,a}\left(\rho\,\sigma_{ij}\right)_{,j}. \end{cases}$$

$$\sigma_{ij} = 0 \quad + \quad \delta \quad \ll \quad 1 \quad \Longrightarrow \quad \frac{\partial^2\delta}{\partial t^2} + 2\,\frac{\dot{a}}{a}\frac{\partial\delta}{\partial t} \quad = \quad 4\,\pi\,G\,\overline{\rho}\,\delta.$$

$$\delta(\mathbf{x},t) = D_+(t)\delta_+(\mathbf{x}) + D_-(t)\delta_-(\mathbf{x}),$$

$$D_+(t) \quad \propto \quad t^{2/3},$$
$$D_-(t) \quad \propto \quad 1/t.$$

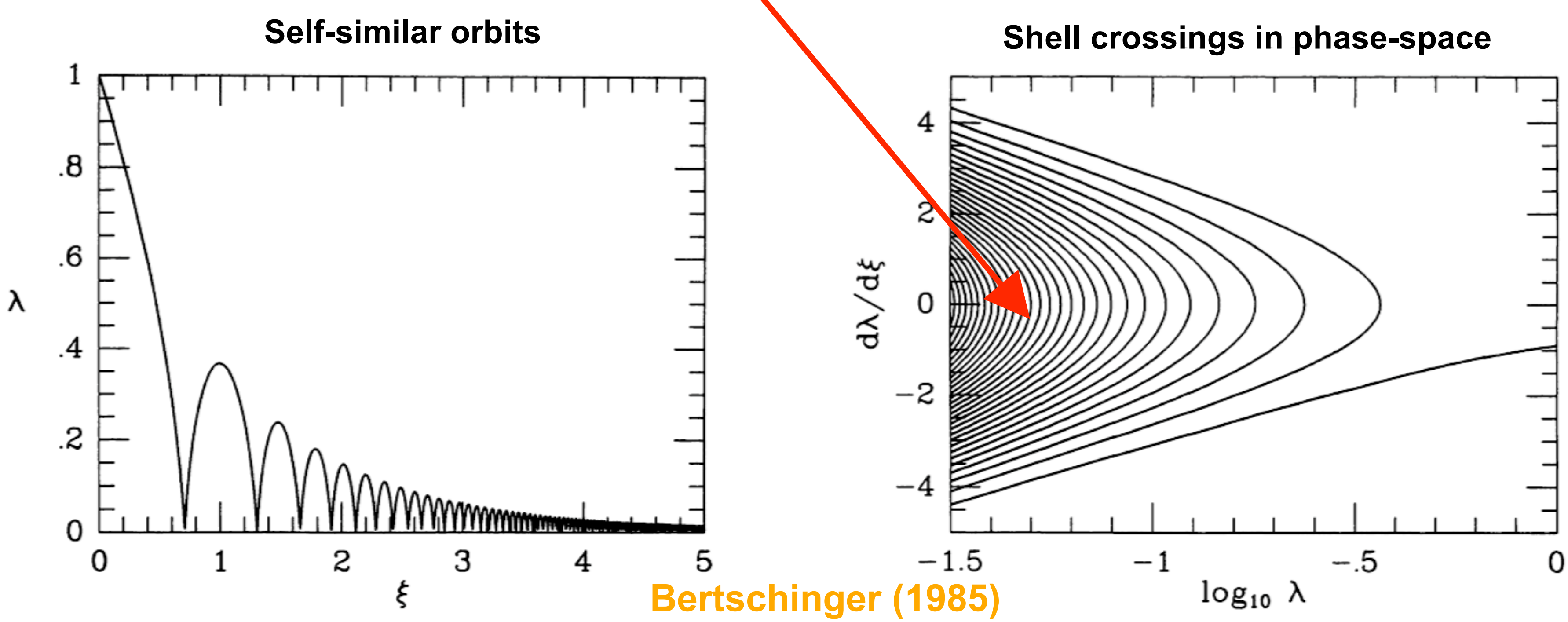
Cours de Francis Bernardeau

Non linear dynamics with the spherical model

$$\ddot{R} = -\frac{G\,M(<R)}{R^2} \qquad \delta_i = \frac{3\,M}{4\pi R_i^3 \rho_0(t)} - 1,$$

Collapse epoch of each shell: $\delta_i\, D_+(t) = \delta_c \text{ avec } \delta_c = 1.69$

Virial equilibrium and stable clustering



Bertschinger (1985)

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Structure formation spans over orders of magnitude in scale

- From a quasi-uniform, well-defined state, gravitational instability generates a population of dense and objects in equilibrium:

- Massive clusters — adiabatic hydrodynamics
  $\Delta \approx 1000$ — $R = 1/10\ R_0$
  thermal equilibrium

- Galaxies — atomic cooling
  $\Delta \approx 10^6$ — $R = 1/100 R_0$
  centrifugal equilibrium

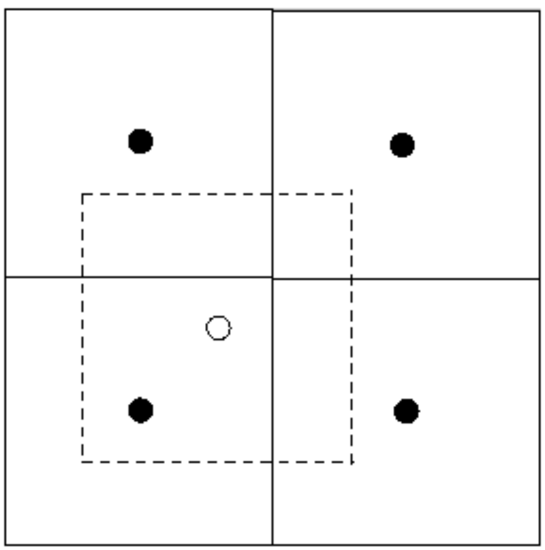
- Molecular clouds — molecular cooling
  $\Delta \approx 10^9$ — $R = 1/1000\ R_0$
  MHD turbulent equilibrium

- Adaptive, multi-scale, numerical methods are mandatory.

The Particle Mesh method in cosmology

N body scheme: $$\frac{d\mathbf{x}_p}{dt} = \mathbf{v}_p \quad \text{and} \quad \frac{d\mathbf{v}_p}{dt} = -\nabla_x \phi$$

- **Computation of the mass density field on a Cartesian grid**
- **Solution of the Poisson's equation for the gravity field**
- **Interpolation of the gravity field from the grid to the particles**



"Cloud-In-Cell" interpolation

Use of a symplectic time integrator:

$$\mathbf{v}_p^{n+1/2} = \mathbf{v}_p^n - \nabla\phi^n \Delta t^n/2$$
$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \mathbf{v}_p^{n+1/2}\Delta t^n$$
$$\mathbf{v}_p^{n+1} = \mathbf{v}_p^{n+1/2} - \nabla\phi^{n+1}\Delta t^n/2$$

Hockney & Eastwood (1981)

RAMSES : a 3D N-body + Hydro Adaptive Mesh Refinement code

- Tree-based AMR (octree structure) : the cartesian mesh is recursively refined *on a cell by cell basis*.
- Full connectivity : each "oct" have direct access to neighboring parent cells and to children "octs". (memory overhead : 2 integers per cell).

→ Optimize the mesh adaptivity to complex geometries, but CPU overhead can be as large as 50%.

N body module :	Particle-Mesh method on AMR grids (similar to the ART code).
	Poisson equation solved using Conjugate Gradient and Multigrid.
Hydro module :	*Unsplit* second order Godunov method : Riemann solver with
	piecewise linear reconstruction (option : MUSCL or PLMDE).
Time integration :	Single time step or W cycle (fine levels subcycling)
Other	Cooling & UV heating, Zoom simulation technology
	MPI based parallel implementation → *Space Filling Curves*

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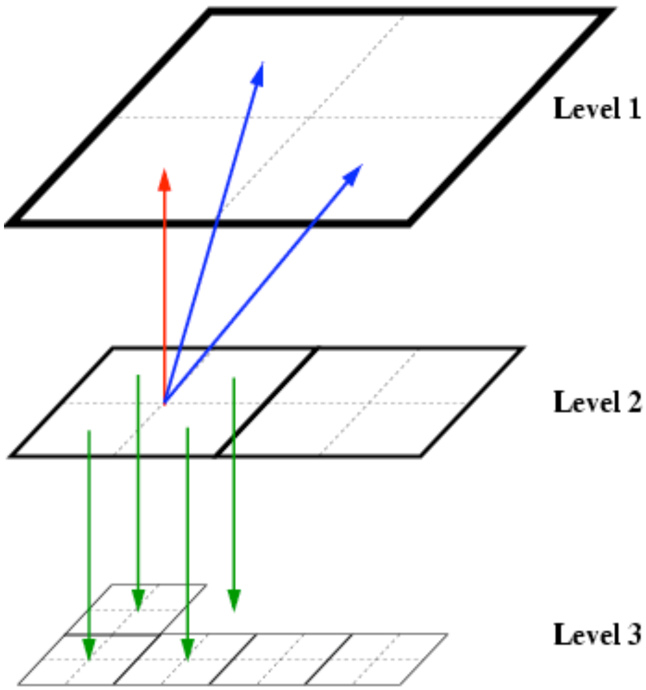
Data structure in RAMSES

Tree based : "Fully Threaded Tree" (Khokhlov 1998)



Basic Cartesian mesh

Recursive refinement on a *cell by cell basis*.

Fundamental objects : small grids of 8 cells or `oct`.

`octs` in the mesh are organized in a *linked list* for each level of refinement.

Full connectivity : each `oct` points towards

→ its "parent" cell

→ its 6 neighboring "parent" cells

→ its 8 "children" `octs`

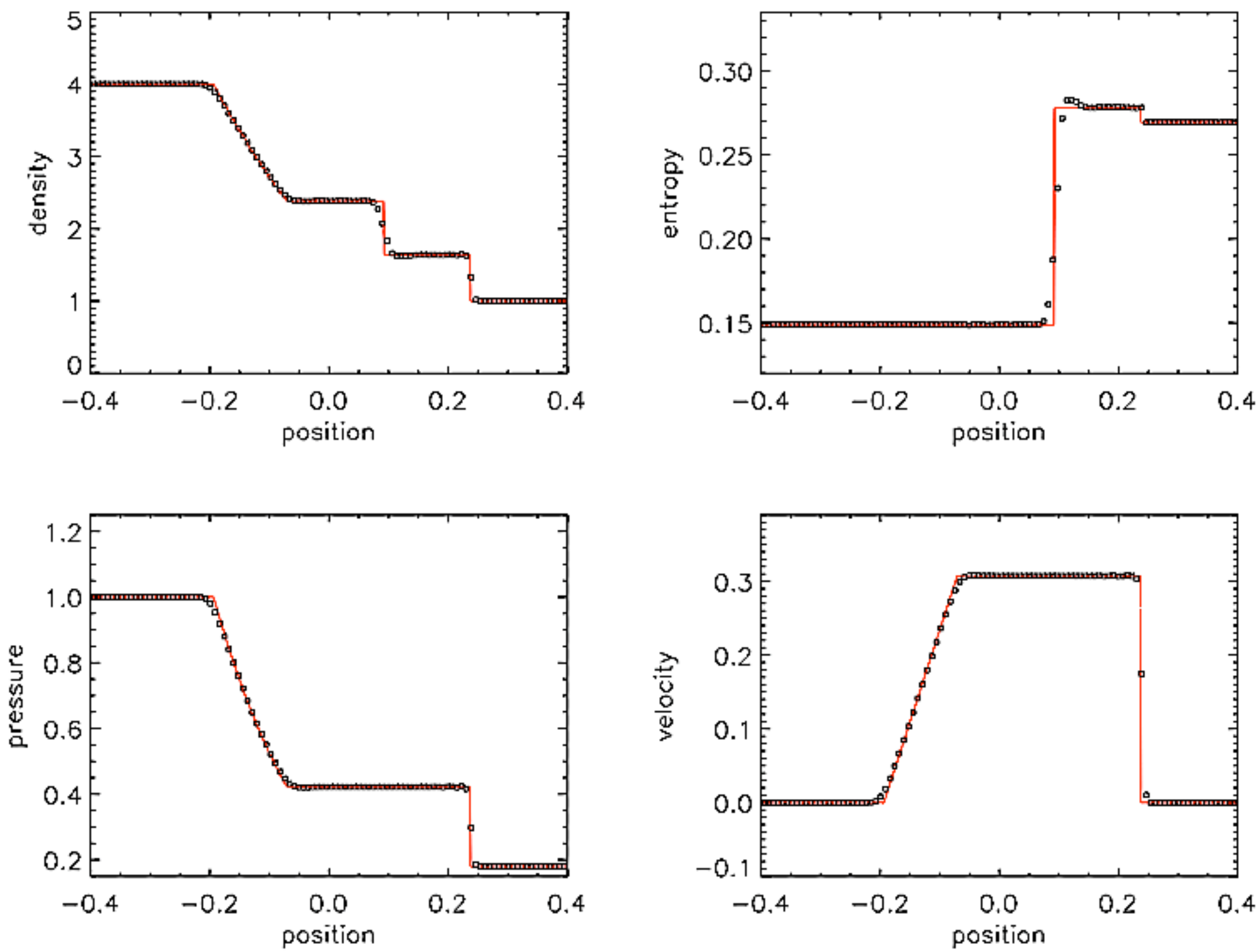
2 disctinct types of cell :

- "leaf" cell or *active*.
- "split" cell *passive*.

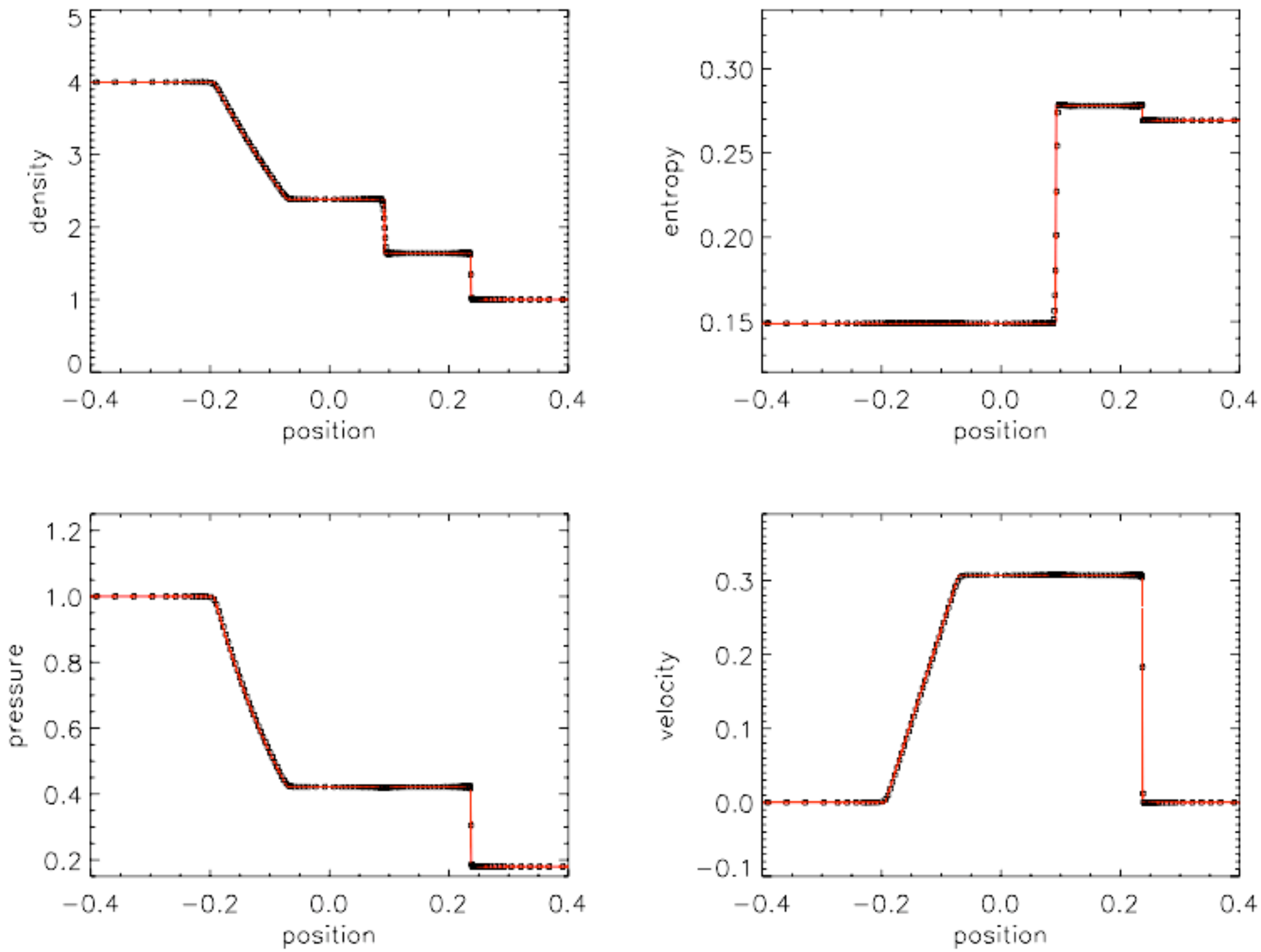
Memory overhead :

- tree management : 2 integers per cell.
- passive cells : 14% in 3D, 33% in 2D, 50% in 1D

Shock-tube test with Eulerian grid



Shock tube test with AMR grid



Rayleigh-Taylor instability : refining surface discontinuities

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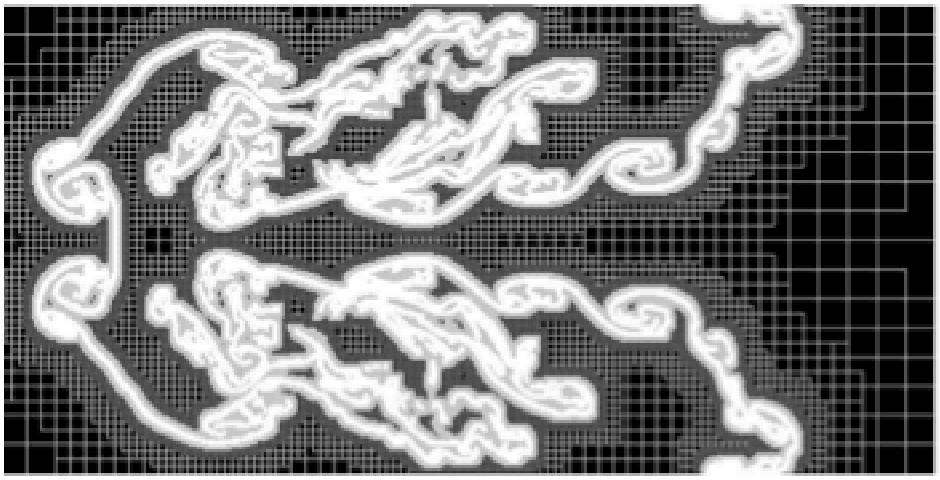
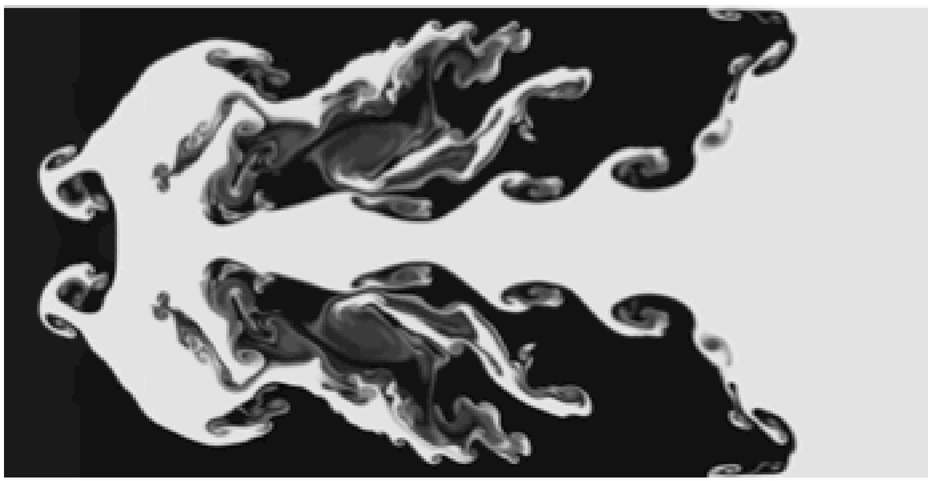




Maximum numerical dissipation occurs at the 2 fluids interface.

The optimal refinement strategy is based on density gradients.

The number of required cells is directly related to the *fractal exponent* n of the 2D surface.

$$N_{cell} \propto (\Delta x)^{-n}$$

Cosmological simulations with AMR

Particle-Mesh on AMR grids:
Cloud size equal to the local mesh spacing

Poisson solver on the coarse grid
Multigrid or FFT

Poisson solver on the fine grids
Conjugate Gradient
Interpolation to get Dirichlet boundary conditions
One way recursive scheme

Quasi-Lagrangian mesh evolution
roughly constant number of particles per cell

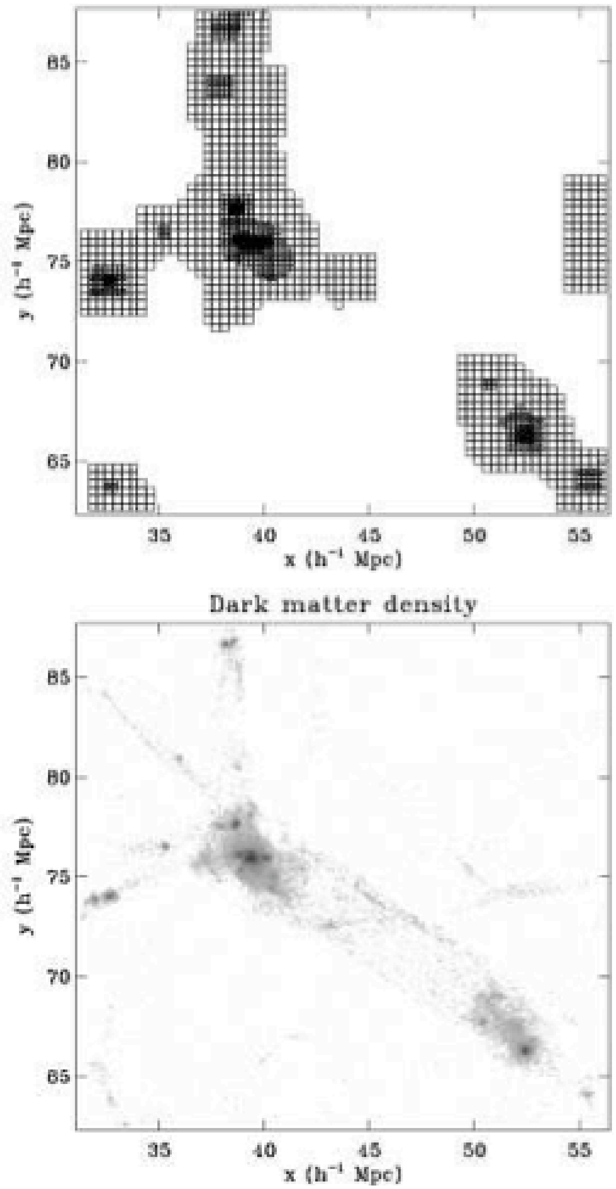
Compute for each cell

$$n = \frac{\rho_{DM}}{m_{DM}} + \frac{\rho_{gas}}{m_{gas}} + \frac{\rho_*}{m_*}$$

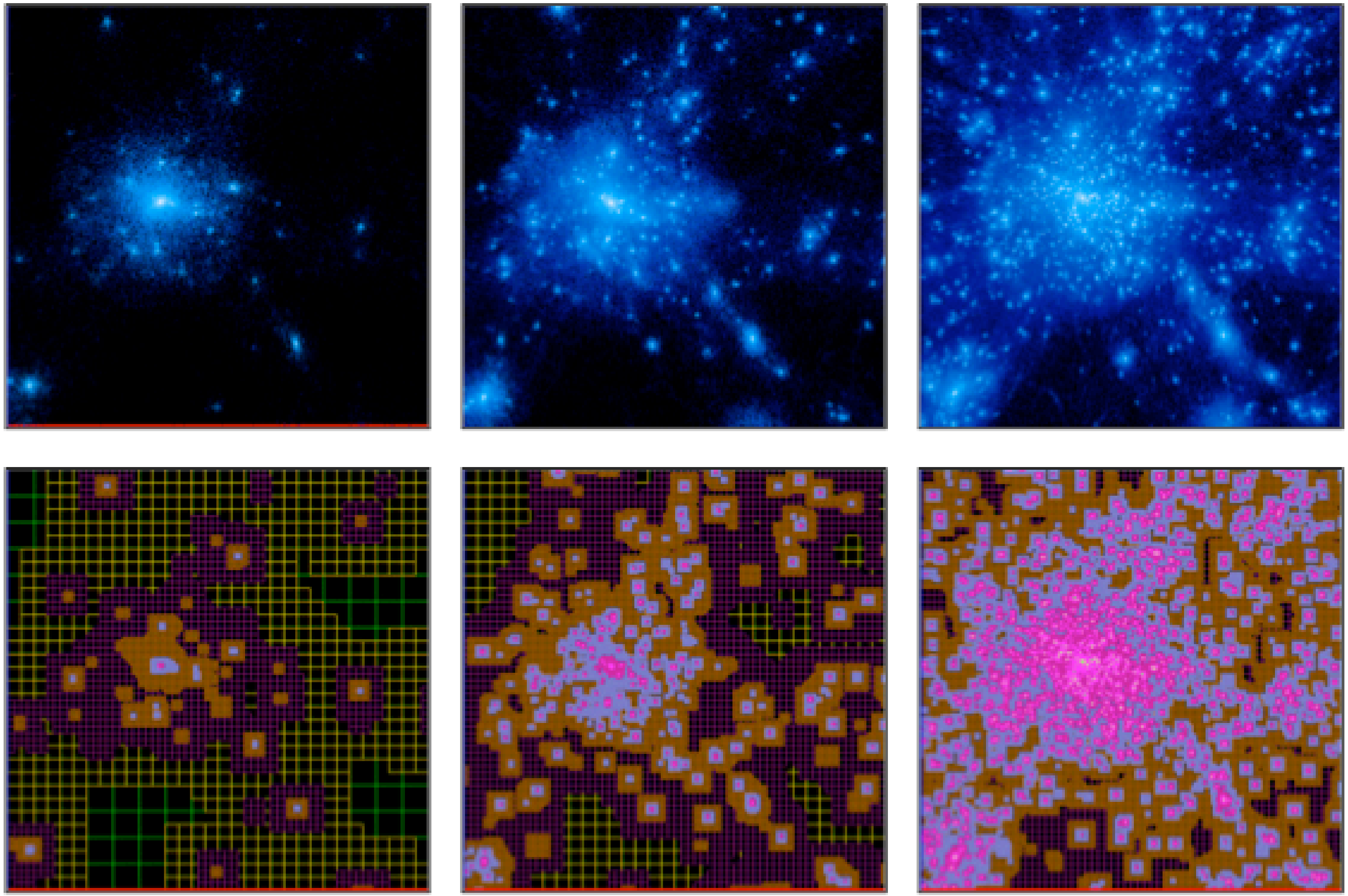
Trigger new refinement when n > 10-40 particles

Fractal dimension close to 1.5 at large scale and is les than 1 at small scales.

Teyssier (A&A 2002)



A Coma-like cluster with increasing resolution : towards galactic scales ?



$N \simeq 5 \times 10^4$ $N \simeq 4 \times 10^5$ $N \simeq 3 \times 10^6$

Image size 12 h^{-1} Mpc

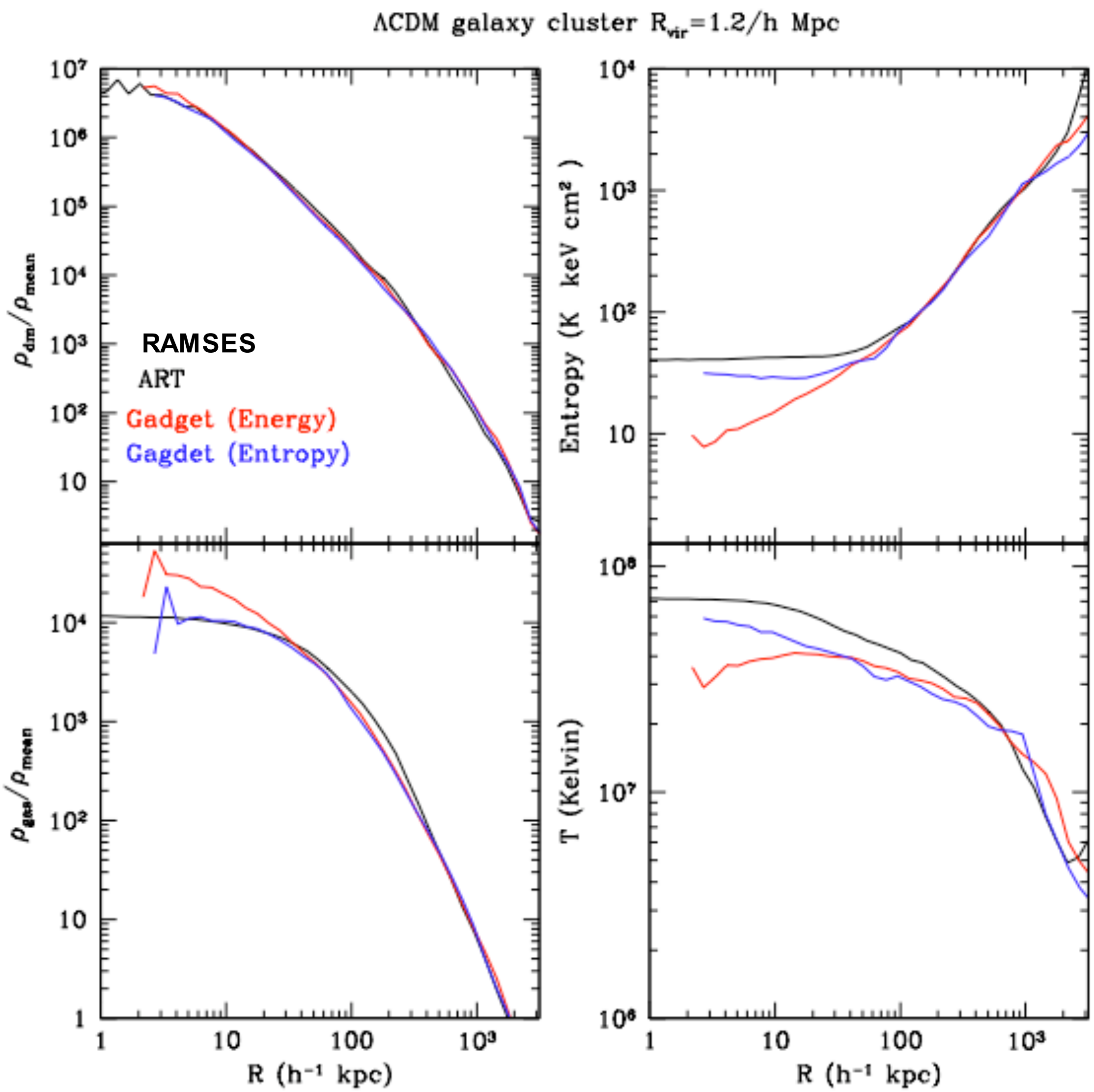
Non-linear dynamics and the halo model

Internal structure of dark matter halos

Compare modern SPH and AMR schemes with N body + hydrodynamics evolution

Is this really correct ?

Impact of numerical dissipation ?



Analytical model for the internal structure of cosmological halos

Dark matter particles follow the NFW profile (Navarro, Frenk & White 97)

$$M(<r) = 4\pi\rho_s r_s^3 \left(\ln(1+x) - \frac{x}{1+x} \right) \qquad r/r_s = x$$

Baryons fluid is considered as a polytrope in hydrostatic equilibrium

$$P \propto \rho^\Gamma \qquad \frac{1}{\rho}\frac{\partial P}{\partial r} = -\frac{GM(<r)}{r^2}$$

Analytical solution for vanishing pressure at infinity

$$T(r) = T_0 \frac{\ln(1+x)}{x} \qquad \frac{k_B T_0}{\mu m_p} = 4\pi G \rho_s r_s^2 \frac{\Gamma - 1}{\Gamma}$$

$$T(r) = T_0 \left(\frac{\rho(r)}{\rho_0} \right)^{\Gamma - 1}$$

Two unknowns: Γ and ρ_0

Suto *et al.* 1998; Komatsu & Seljak 2001

Analytical model for the internal structure of cosmological halos

Assume that hydrostatic equilibrium is valid only at high overdensity. At low density, the gas profile is parallel to the dark matter one.

$$\Delta_{eq} \simeq 1000\bar{\rho}$$

Connect smoothly the two regions at

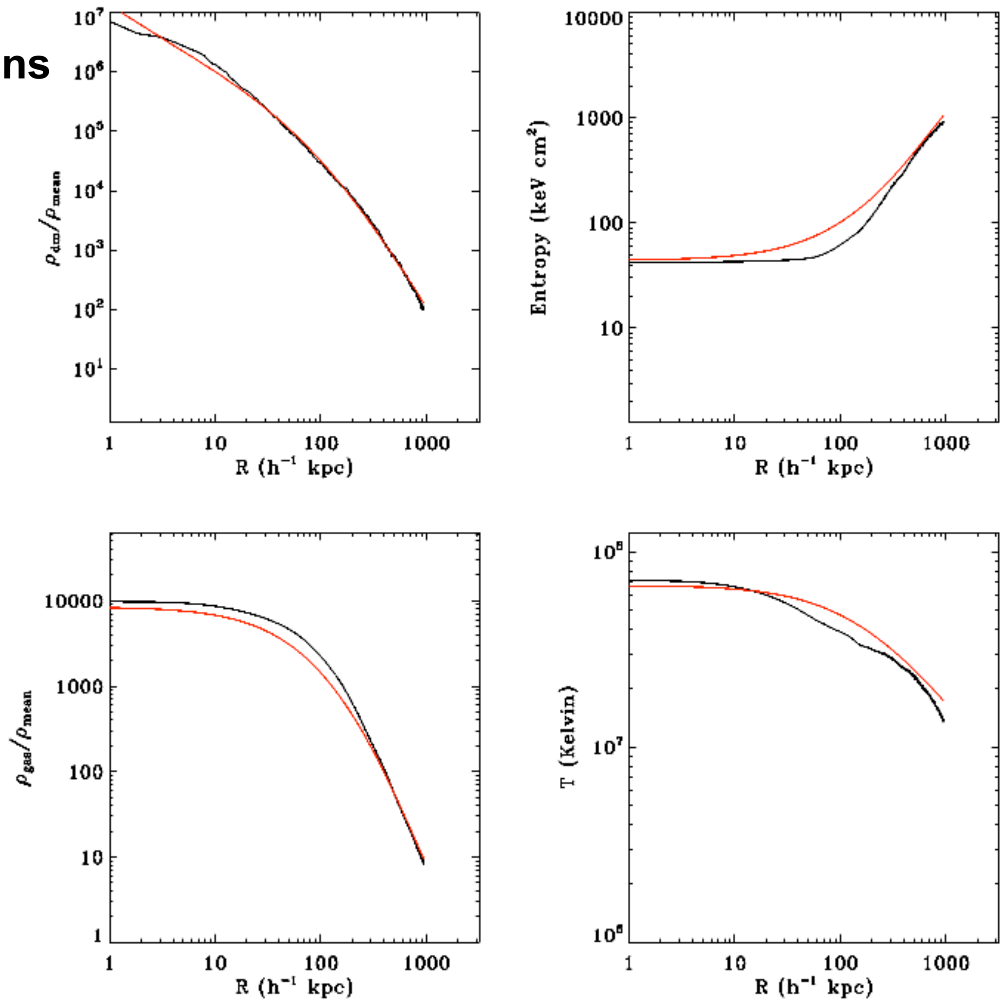
$$x_{eq} \simeq \sqrt{5}c,$$

One obtains :

$$\Gamma = 1 + \frac{(1+x_{eq})\ln(1+x_{eq}) - x_{eq}}{(1+3x_{eq})\ln(1+x_{eq})}$$

Finally determine ρ_0 by normalizing to the baryons total mass

Dark matter and gas halos are a "2 parameter" family (M_{200} and c)



Dark matter halos statistics: the mass function

What is the number density of halos on a given mass range ?

Generalized Press & Schechter (1974) theory: $$\Phi(M) = -\frac{\rho_0}{M}\frac{d}{dM}\left(\frac{F(>M)}{F_0}\right)$$

$$F(\geq M) = \int P_R\left(x^j\right) s\left(x^j\right) dx^1 \cdots dx^n$$

s: selection function on the initial conditions smoothed at scale R

s = 1 if $a_c\left(x^j\right) \leq 1$

s=0 otherwise

A simple model: spherical collapse $$1 + z_c = \frac{\delta}{\delta_c}$$

Gaussian statistics: $$P_\nu = \frac{1}{\sqrt{2\pi}}e^{-\nu^2/2} \qquad \nu = \delta/\Delta \qquad \Delta^2(R) = \int 4\pi k^2 P(k) W^2(kR) dk$$

Analytical prediction: $$\Phi(M) = \frac{\overline{\rho}}{M}\frac{d\nu_c}{dM}\sqrt{\frac{2}{\pi}}e^{-\nu_c^2/2}.$$

Press & Schechter revisited: more complex dynamical model (ellipsoidal collapse)

Audit,Teyssier & Alimi (1998); Sheth, Mo & Tormen (2001)

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Computing the non-linear power spectrum: the halo model

Idea: described the dark matter density field as a collection of isolated halos.

3 basic ingredients to be determined in advance:

-The halo internal structure: **NFW profile (M_{200} and c)**

$$c(M,z) \simeq g(z)\left[\frac{M}{M_*(z)}\right]^{-h(z)},$$

-The halo mass function: **PS theory**

-The halo linear bias: **Mo & White theory**

$$b(M,z) = 1 + \frac{\nu^2 - 1}{\delta_c}.$$

$$P(k,z) = P_1(k,z) + P_2(k,z),$$

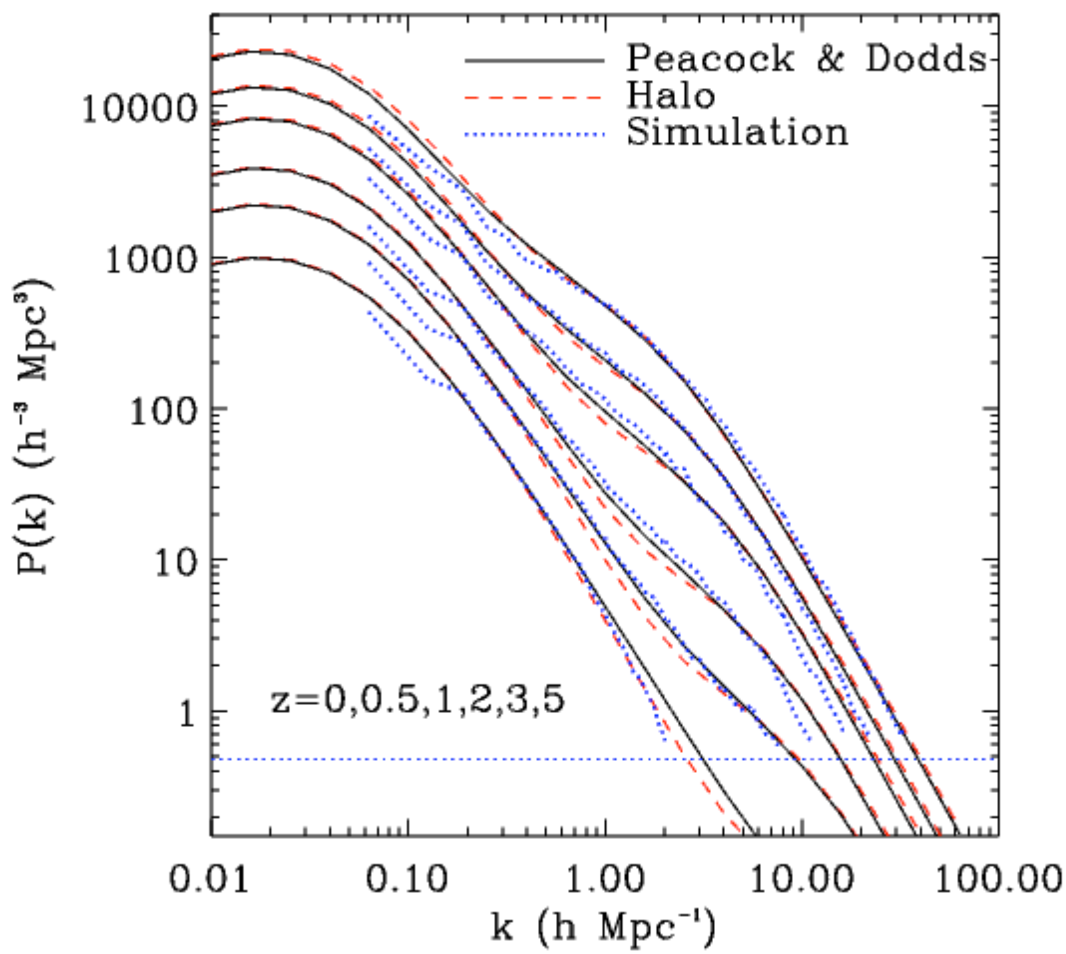
1-halo term:

$$P_1(k) = \int_0^\infty dM \, \frac{dn}{dM}\left[\frac{\tilde{\rho}(k,M)}{\overline{\rho}}\right]^2$$

2-halo term:

$$P_2(k) = \left[\int_0^\infty dM \, \frac{dn}{dM} b(M) \frac{\tilde{\rho}(k,M)}{\overline{\rho}}\right]^2 P_{\rm lin}(k)$$

Computing the non-linear power spectrum: the halo model



Refregier et Teyssier (2002)

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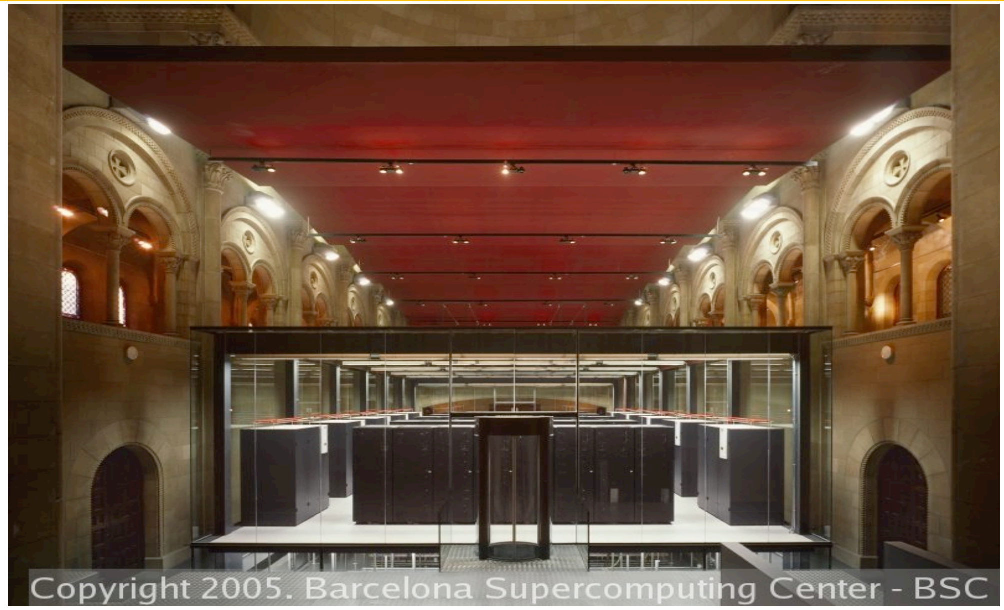
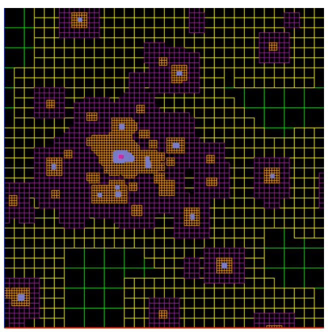
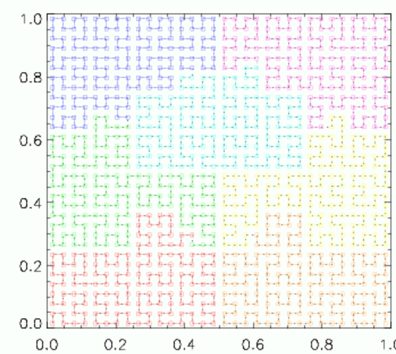
Towards many-billions particles simulations



The Mare Nostrum galaxy formation project in Barcelona



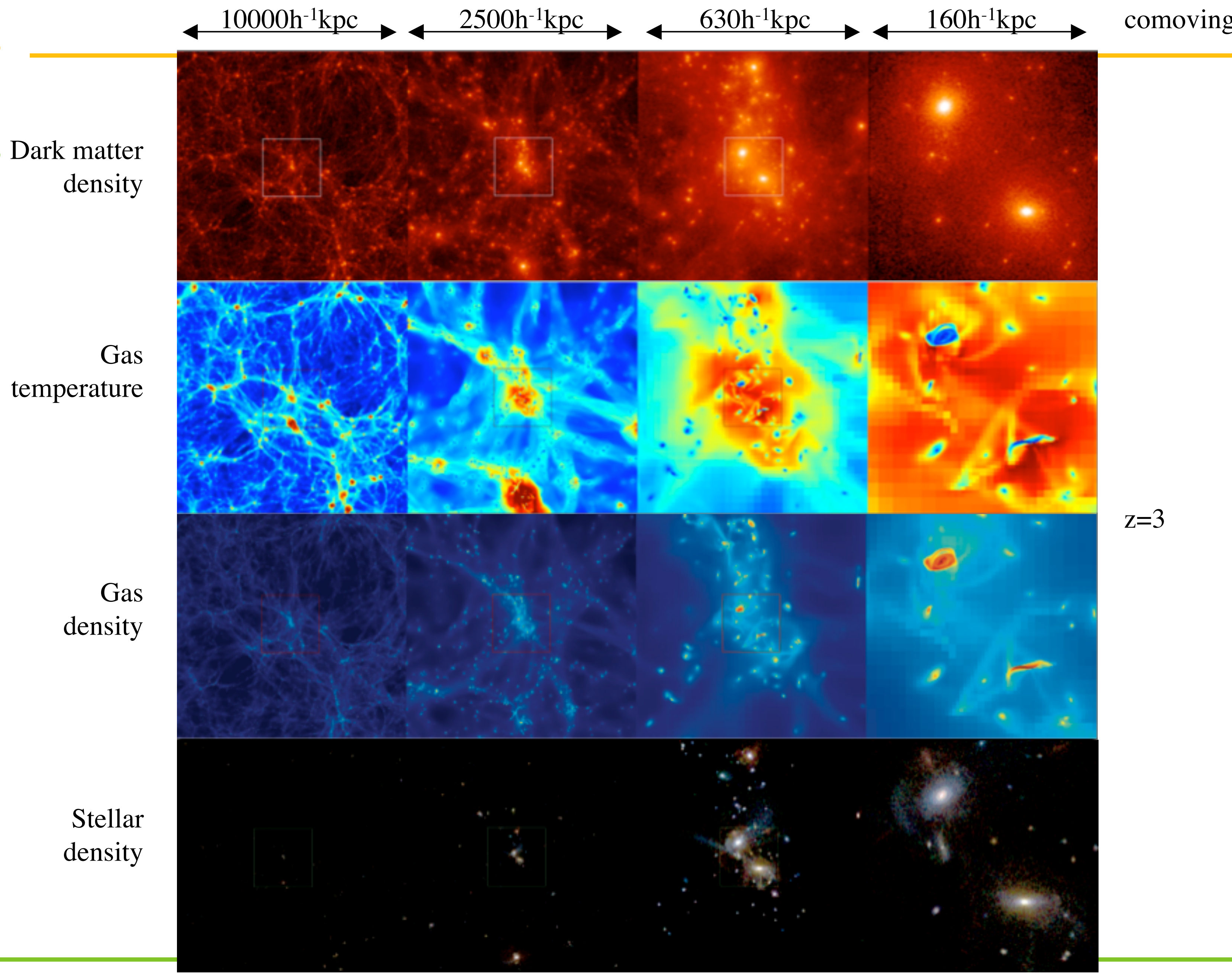
RAMSES
Teyssier, R. A&A, 2002, 385, 337
Tree based AMR code
Unsplit high-order Godunov
Galaxy formation physics
Adaptive domain decomposition using "space filling curve"
MPI and F90



Horizon was selected with 27 other projects to run one extreme application on **Mare Nostrum** in Barcelona SC (5th computer worldwide)
Source: http://www.deisa.org and http://www.top500.org

Present:
Good scalability up to 512 processors and 1024^3 particles
Cooling & UV heating using external spectrum
Star formation and supernovae feedback

Objectives: run RAMSES up to 2048 processors
Goal: 10^{10} AMR cells with self-consistent cooling & UV heating (spectrum & metals)
Metal enrichment using galactic winds


10000h-1kpc
2500h-1kpc
630h-1kpc
160h-1kpc
comoving
Dark matter density
Gas temperature
Gas density
Stellar density
z=3

Fin du diaporama