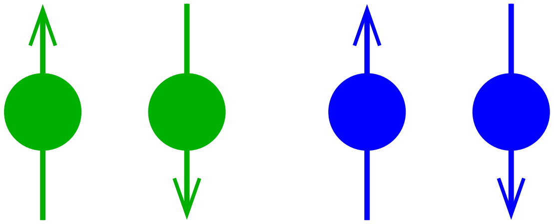
$\alpha-$Particle Condensation in Nuclear systems

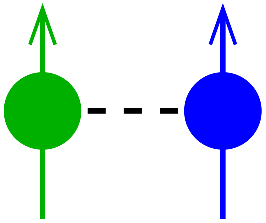
A. Tohsaki, H. Horiuchi, G. Röpke, P. Sch.
T. Yamada and Y. Funaki

- α-condensation in ∞ matter
- ^{8}Be and Hoyle state in ^{12}C*
- α-condensate wave function
- Effective GPE for n.α condensate
- Conclusions, outlook

Clusters important aspect and richness of nuclear systems due to 4 Fermions :

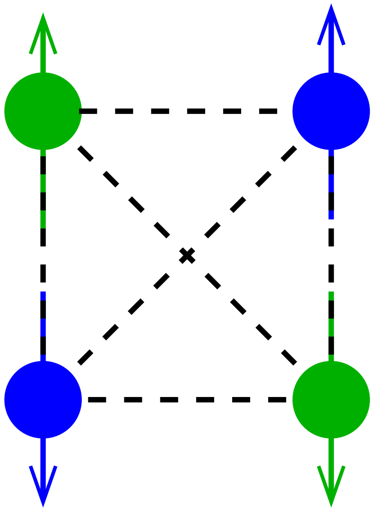


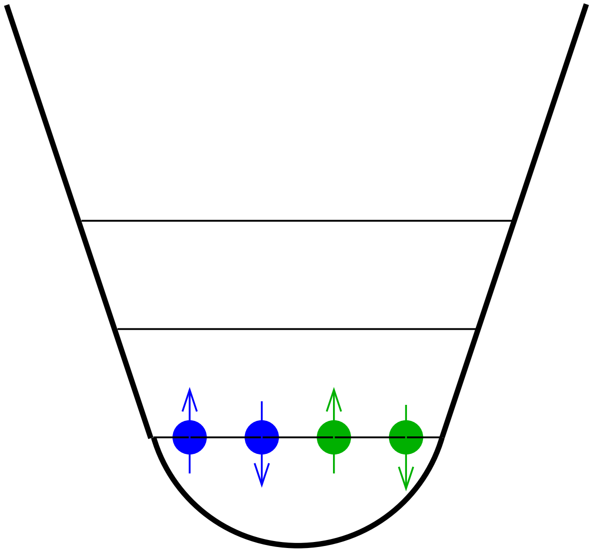
Dimer :



$\frac{E}{A}$ =1 MeV

Quartet :





$\frac{E}{A}$ = 7 MeV, E^{*}= 20 MeV

Proposal :

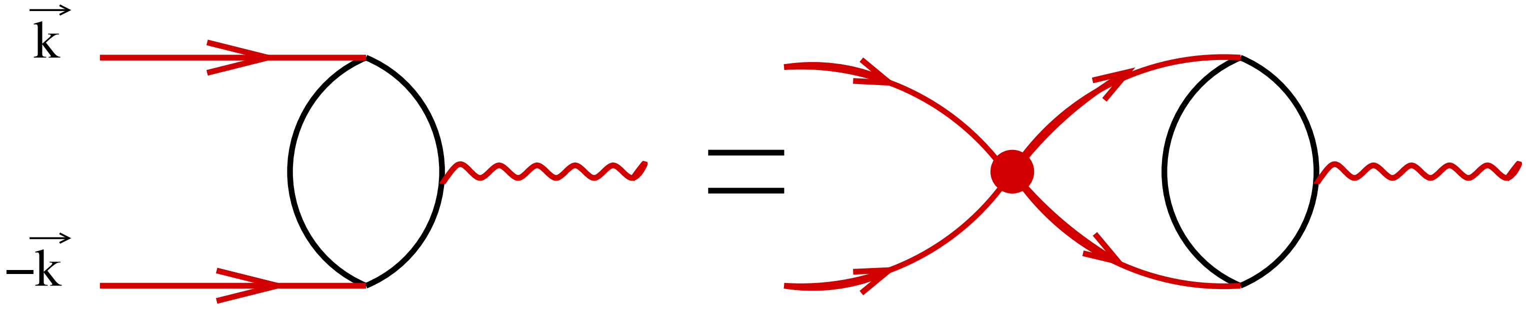
Trapping of 4 different species of Fermionic atoms.

Infinite matter :

- Pair Condensation (nn or pn) $\epsilon_1 = \dfrac{p_1^2}{2m}$
- Thouless criterion for T_c : $f_1 = \dfrac{1}{1 + e^{\frac{(\epsilon_1 - \mu)}{T}}}$

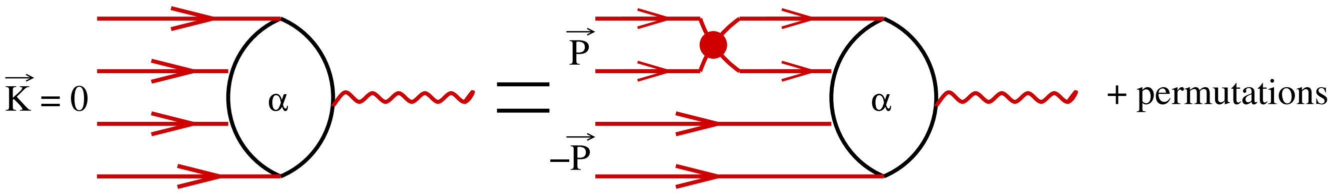
$$\boxed{(2\mu - \epsilon_1 - \epsilon_2)\, \psi_{12} = (1 - f_1 - f_2) \sum_{1'2'} v_{121'2'} \psi_{1'2'}}$$

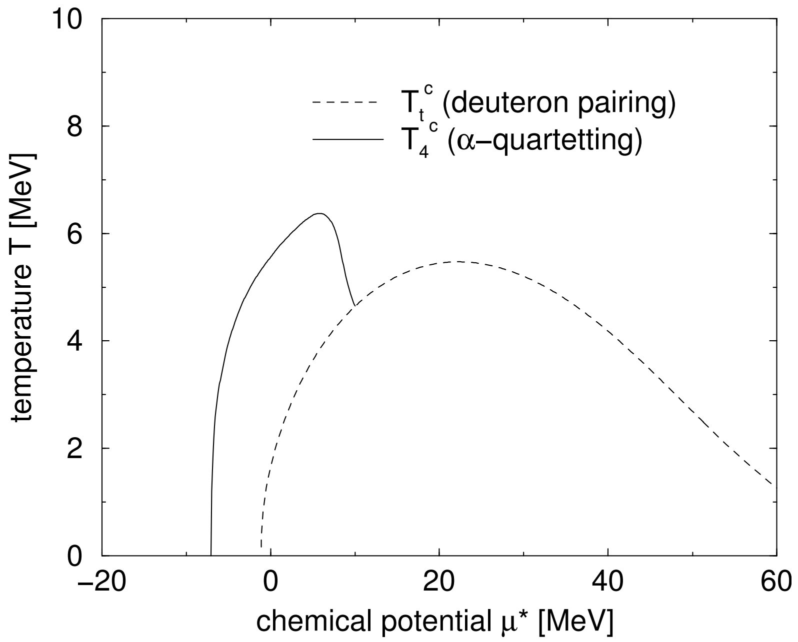
μ chemical-potential and f_1, f_2 Fermi-Dirac at $T = T_c$.

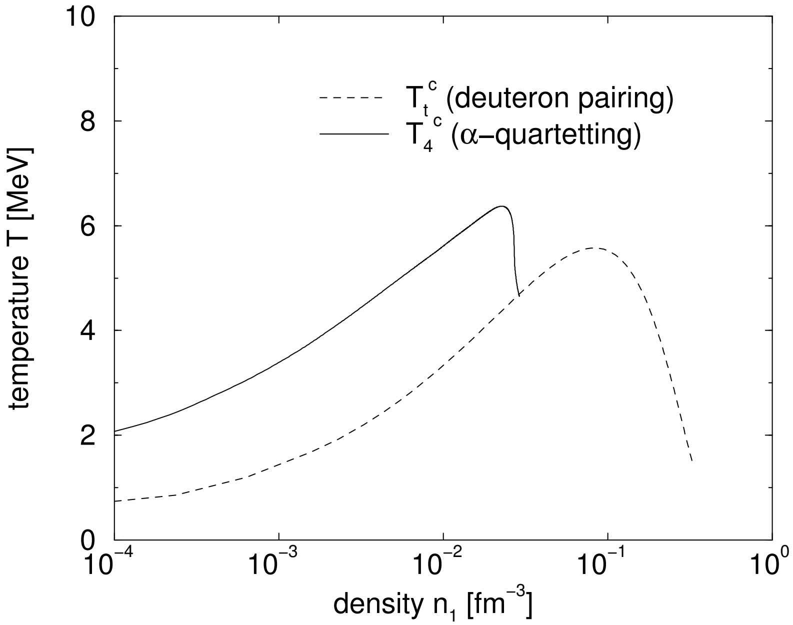


- α-Particle Condensation : G. Röpke, M. Beyer

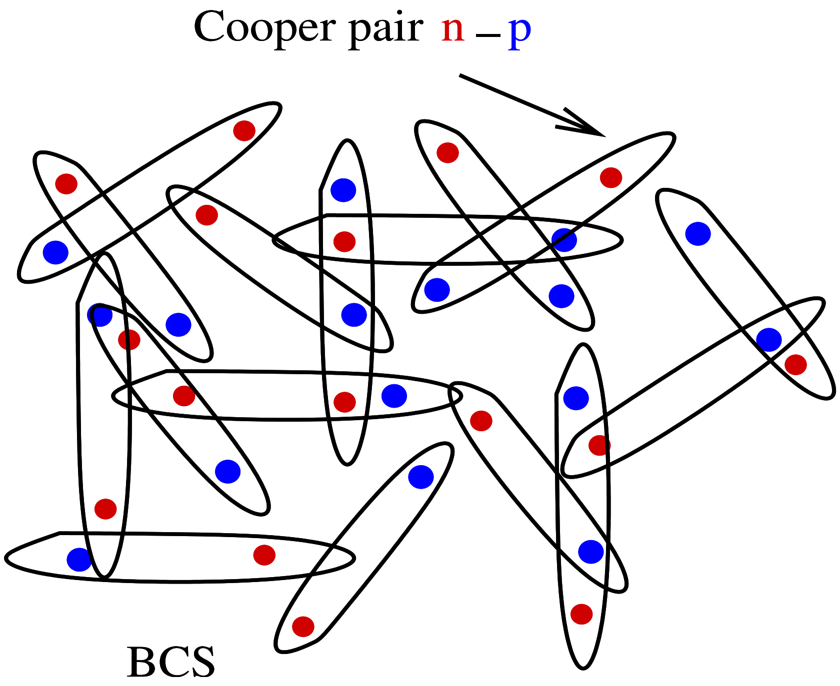
$$(4\mu - \epsilon_1 - \epsilon_2 - \epsilon_3 - \epsilon_4)\,\psi_{1234} = \quad (1 - f_1 - f_2) \sum_{1'2'} v_{121'2'} \psi_{1'2'34} \\ +\text{permutations}$$







α-Condensation only at very low density !



High Density

n–p Cooper pairs

Strongly overlapping

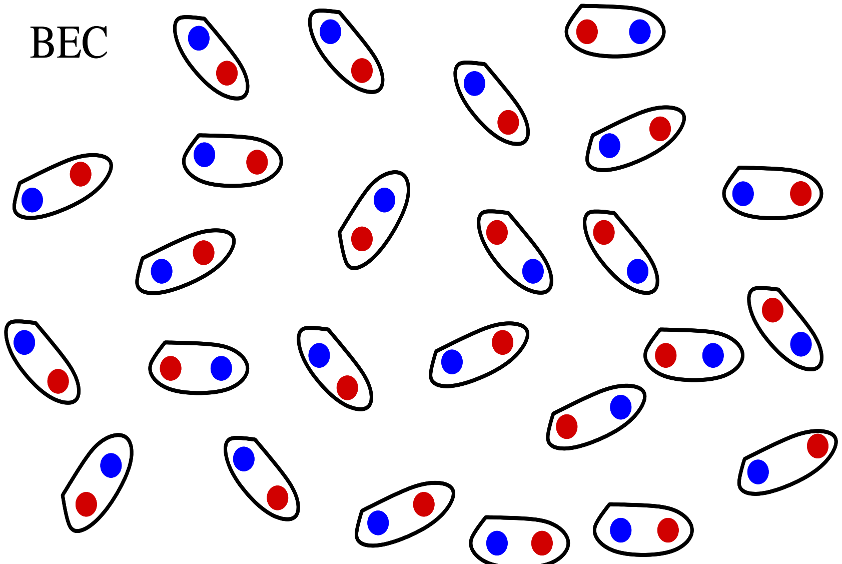
not Bosons

Low density : ↕ smooth transition

BCS

↕

BEC



gas of Deuterons

~ Bosons

Finite nuclei ?

Exact ^{8}Be :

Density : $\frac{\rho_0}{3}$

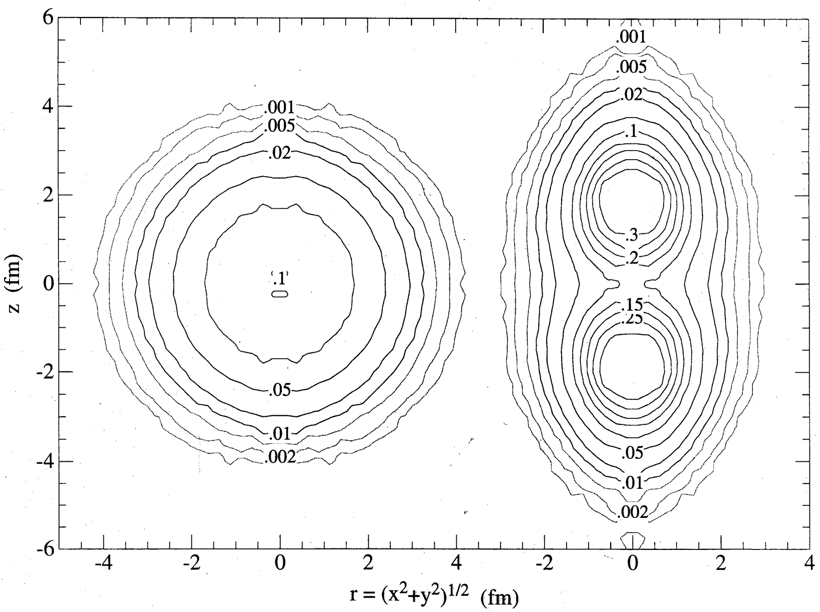
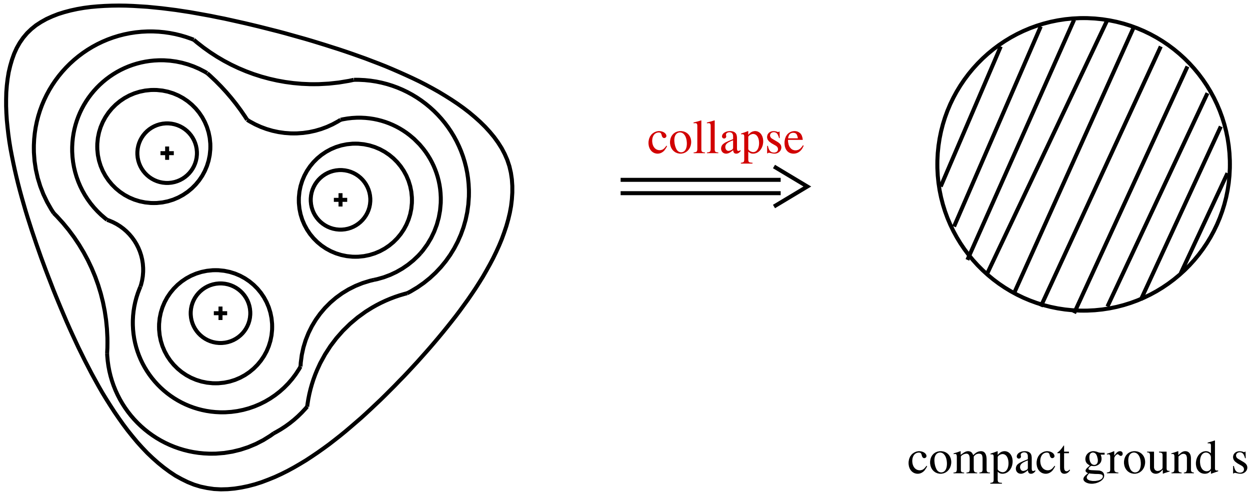


Fig. 15 (Wiringa, et al.)

3 rd α–particle



collapse

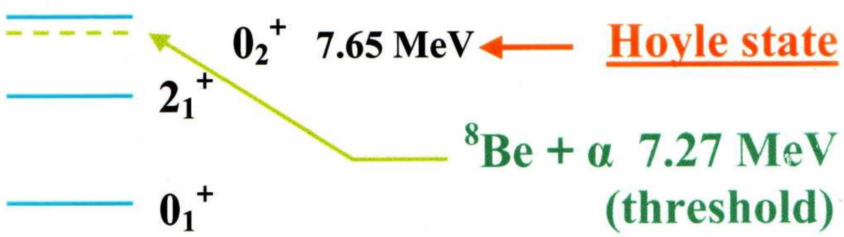
Fermi gas

^{12}C

compact ground state V/$_3$

V

Does a dilute 3α $^{12}C^*$ state exist ?
Similar to $^8Be + \alpha$?



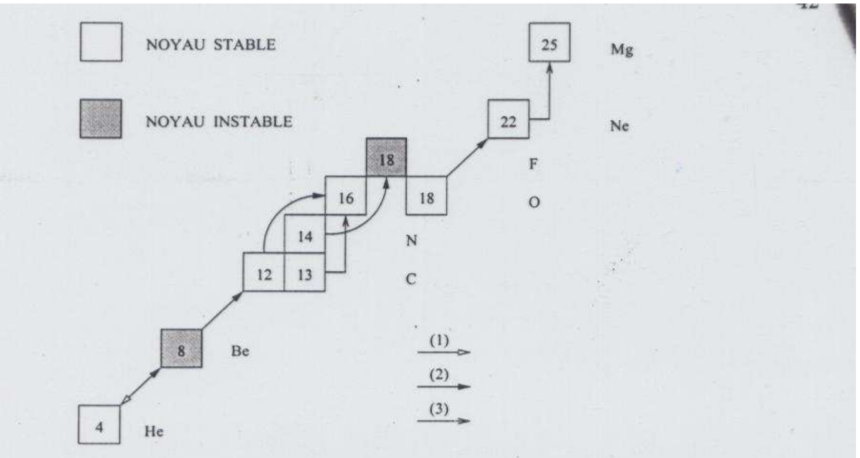
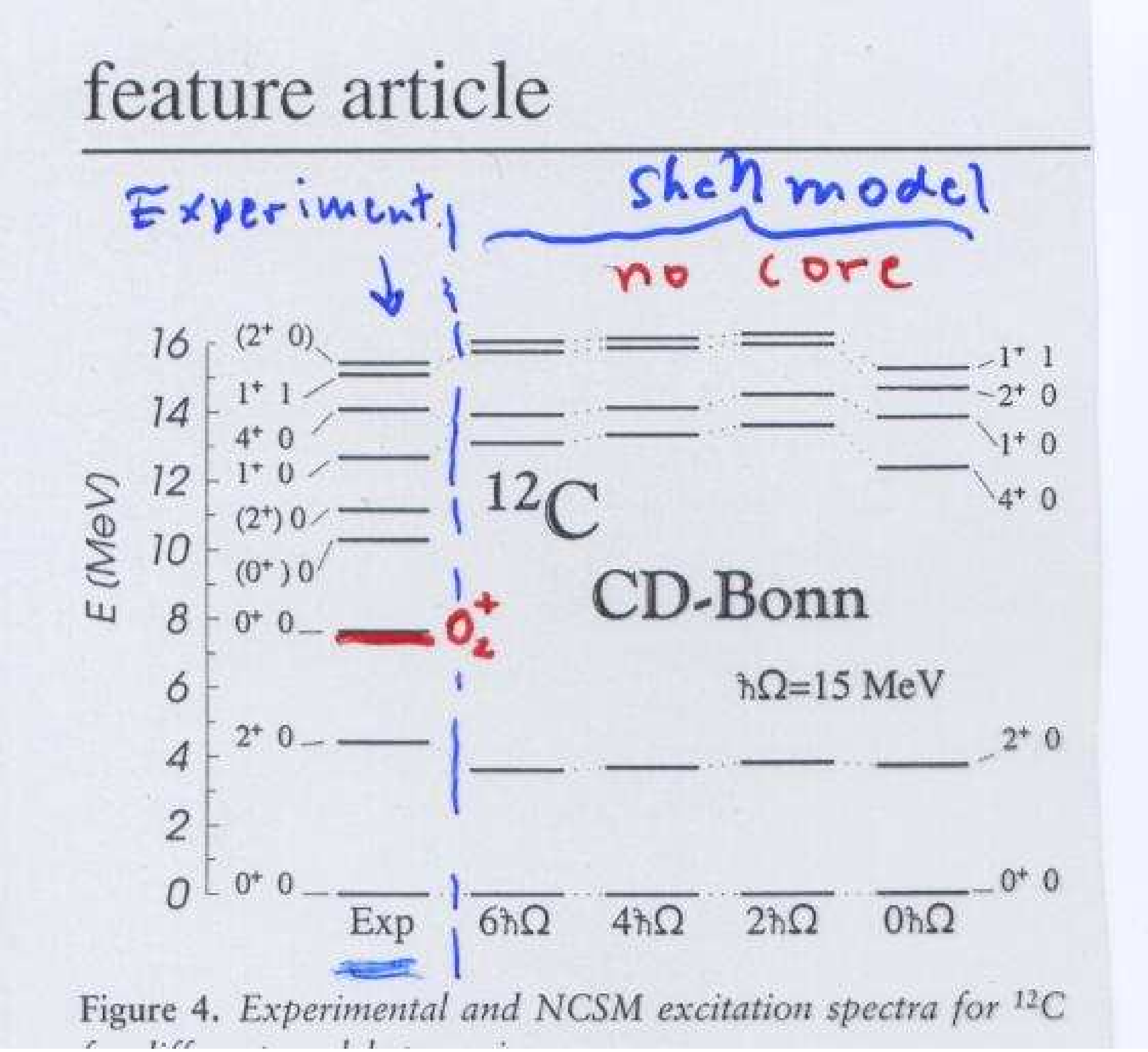


Figure 26. Réactions principales impliquées dans la combustion non explosive de He. Les flèches de type (1), (2) et (3) se rapportent respectivement à des réactions (γ, α), (α, γ), et (α, n)

At $T = 10^8 K$ helium burning
thermal equilibrium

$$\alpha + \alpha + \alpha \;\rightarrow\; {}^8Be \;\rightarrow\; \alpha \;\rightarrow\; {}^{12}C^* \quad O_2^+$$

O_2^+ : dilute 3α state hypothesis !



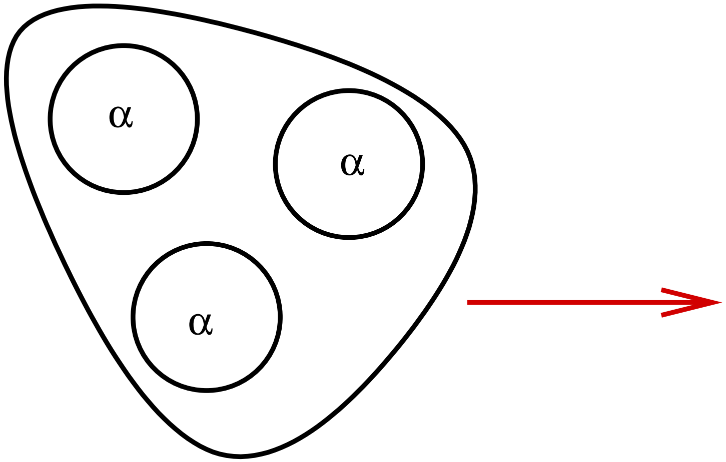
feature article

Figure 4. *Experimental and NCSM excitation spectra for* ^{12}C

it seems impossible to get Hoyle state from shell model calculation !

45 MeV B. Barret

If O_2^+ in^{12}C dilute $\alpha -$ state

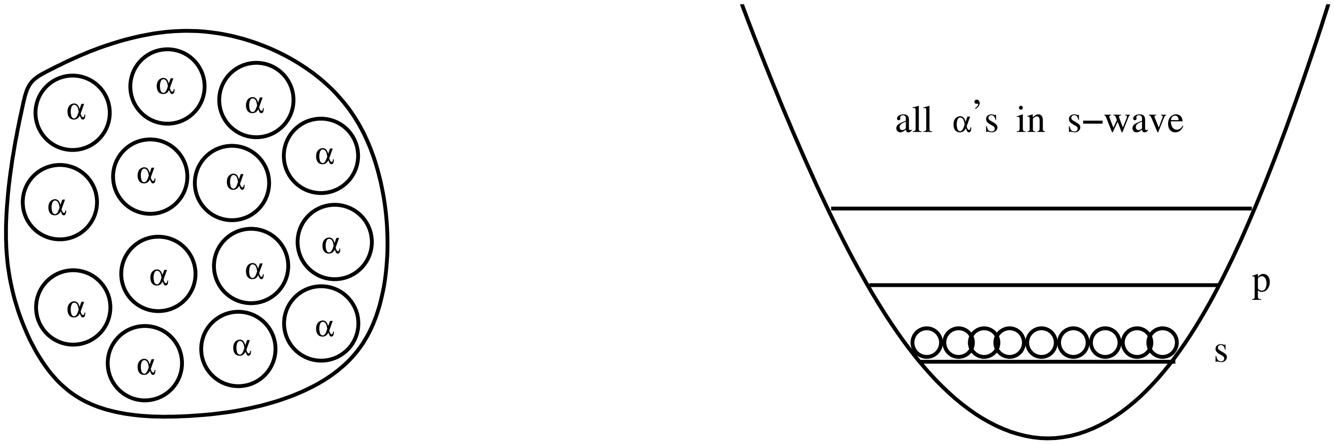


then α-condensate

infinite matter $\rho_{\text{crit}} \sim \frac{\rho_0}{3}$

Conjecture

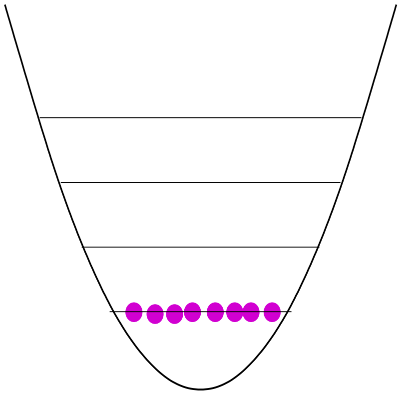
all n.α nuclei possess exited nα condensed state

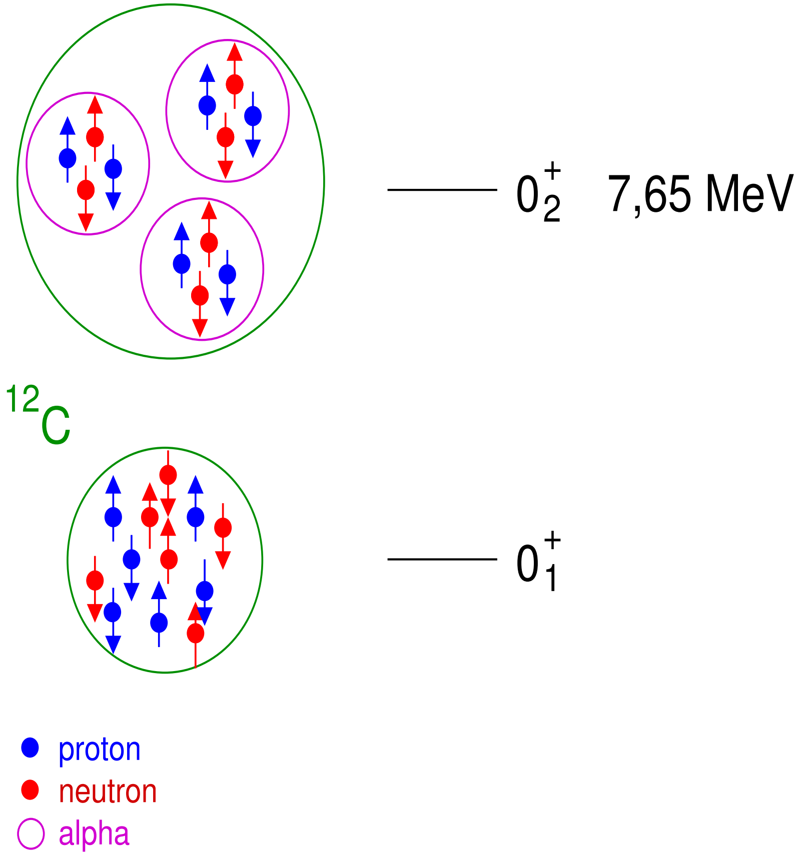


Analogy with atoms in traps ! $\rho(r) = N|\phi_0(r)|^2$ $N = 10^6$

Bosons

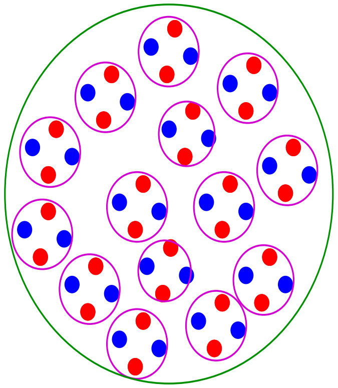
Back to nuclei



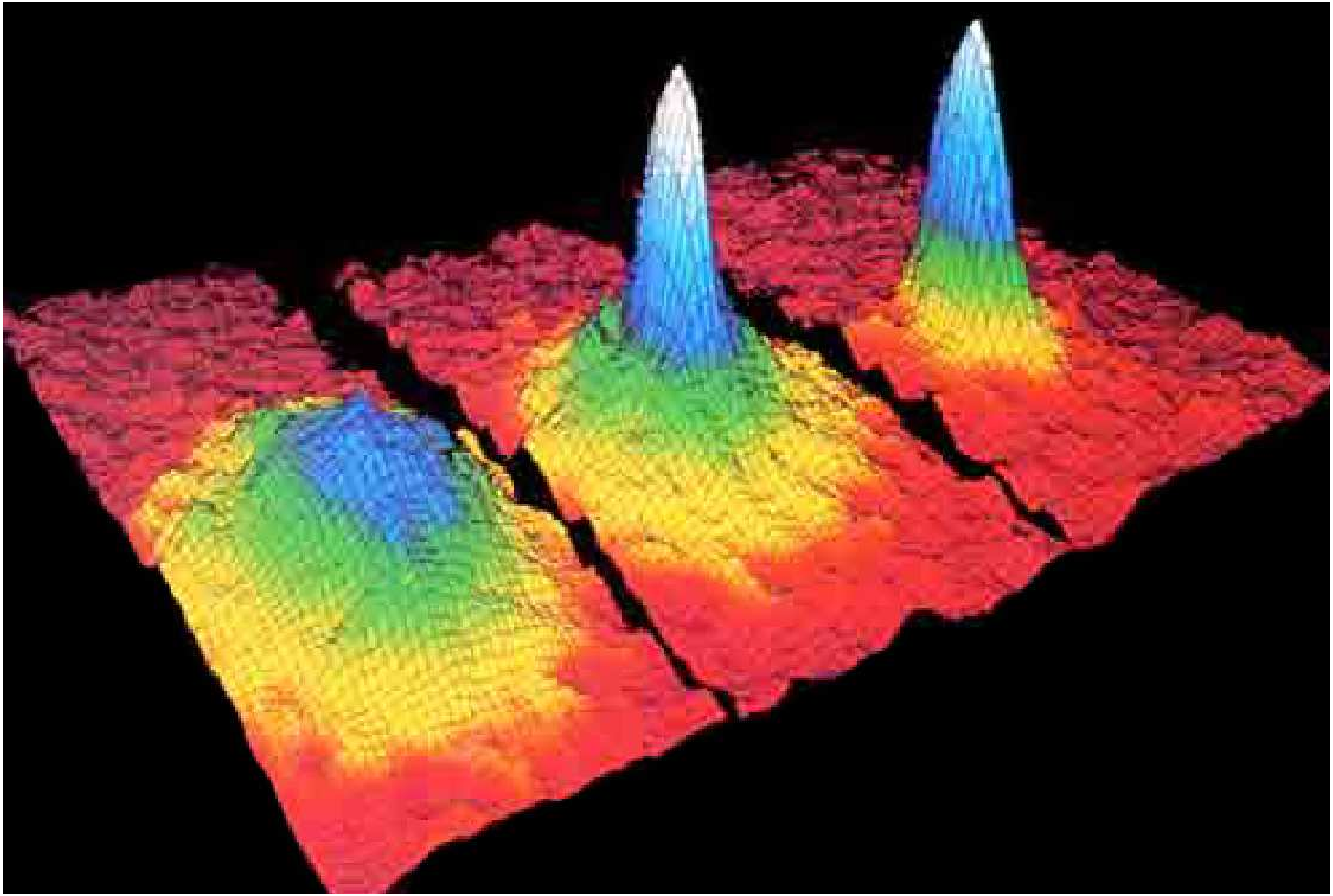


many α's

→ condensate



strong cluster
phenomena in
lighter nuclei



Theoretical Description

Ideal Bose condensate : $|0\rangle = b_0^\dagger b_0^\dagger \cdots b_0^\dagger |vac\rangle$

α-particle condensate : $|\Phi_{\alpha C}\rangle = C_\alpha^\dagger C_\alpha^\dagger \cdots C_\alpha^\dagger |vac\rangle$

In r-space :

$$\langle \vec{r}_1, \vec{r}_2, \cdots, \vec{r}_{4n} | \Phi_{\alpha \mathrm{C}} \rangle = \mathcal{A}\left\{ \Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) \Phi(\vec{r}_5, \vec{r}_6, \vec{r}_7, \vec{r}_8) \cdots \Phi(\vec{r}_{4n-3}, \vec{r}_{4n-2}, \vec{r}_{4n-1}, \vec{r}_{4n}) \right\}$$

In comparison with pairing :

$$\langle \vec{r}_1, \vec{r}_2, \cdots | \mathrm{BCS} \rangle = \mathcal{A}\left\{ \Phi(\vec{r}_1, \vec{r}_2) \Phi(\vec{r}_3, \vec{r}_4) \cdots \right\}$$

Variational ansatz for $\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4)$:

$$\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = e^{-\frac{2}{B^2}\vec{R}^2} \phi_\alpha(\vec{r}_i - \vec{r}_j)$$

Center of mass : $\vec{R} = \frac{1}{4}\left(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4\right)$

Intrinsic α-wave function :

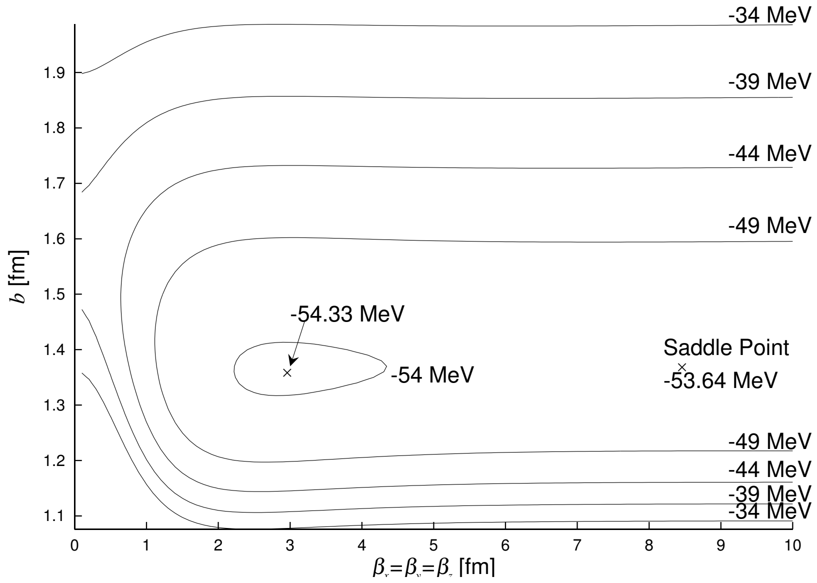
$$\phi_\alpha\left(\vec{r}_i - \vec{r}_j\right) = e^{-\frac{1}{8b^2}\left\{\left(\vec{r}_4 - \vec{r}_1\right)^2 + \left(\vec{r}_4 - \vec{r}_2\right)^2 + \left(\vec{r}_4 - \vec{r}_3\right)^2 + \cdots\right\}}$$

Two variational parameters : $B,\ b$

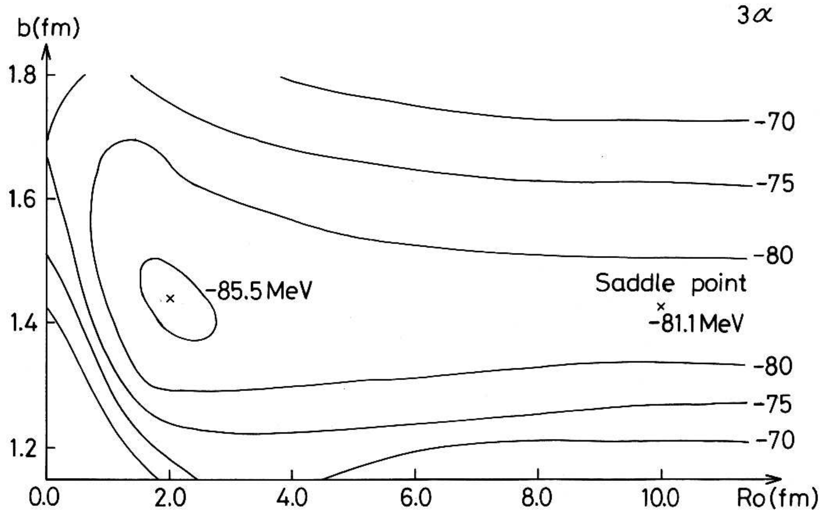
Two limits : $B = b$ $\quad \left|\Phi_{\alpha\mathrm{C}}\right\rangle =$ Slater determinant

$B \gg b$ $\quad \left|\Phi_{\alpha\mathrm{C}}\right\rangle =$ gas of independent α-particles

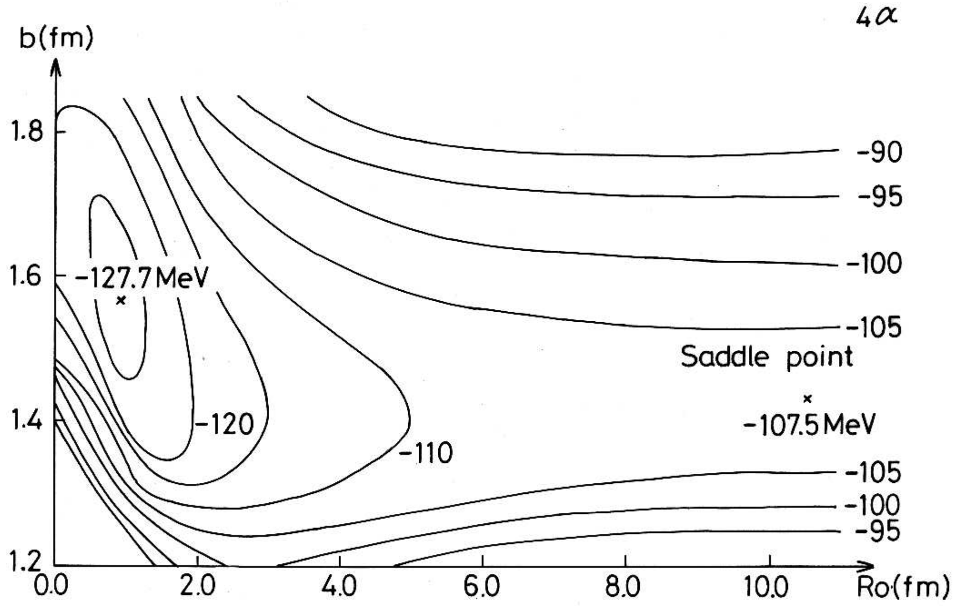
Two dimensional surface : $E\left(B,b\right) = \dfrac{\left\langle\Phi_{\alpha\mathrm{C}}\middle|H\middle|\Phi_{\alpha\mathrm{C}}\right\rangle}{\left\langle\Phi_{\alpha\mathrm{C}}\middle|\Phi_{\alpha\mathrm{C}}\right\rangle}$


-34 MeV
-39 MeV
-44 MeV
-49 MeV
-54.33 MeV
-54 MeV
Saddle Point
-53.64 MeV
-49 MeV
-44 MeV
-39 MeV
-34 MeV
b [fm]
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
0
1
2
3
4
5
6
7
8
9
10
$\beta_x=\beta_y=\beta_z$ [fm]

^{8}B


3α
b(fm)
1.8
1.6
1.4
1.2
-70
-75
-80
-85.5 MeV
Saddle point
-81.1 MeV
-80
-75
-70
0.0
2.0
4.0
6.0
8.0
10.0
Ro(fm)

^{12}C


4α
b(fm)
1.8
1.6
1.4
1.2
-90
-95
-100
-105
-127.7 MeV
-120
-110
Saddle point
-107.5 MeV
-105
-100
-95
0.0
2.0
4.0
6.0
8.0
10.0
Ro(fm)

^{16}O

Hamiltonian :

$$\underset{}{H} = \underset{\text{Kin. energy}}{T} + \underset{\text{Gaussien}}{V_{N-N}} + \underset{\text{Coulomb}}{V_C} + \underset{\text{Gaussian}}{V_{N-N-N}}$$

Quantization of energy surface $E(B,b)$:

Force : A. Tohsaki ~ 1990 no adjustable parameters !

Hill-Wheeler : $|\psi\rangle = \sum_B f_B |\Phi_{\alpha C}(B)\rangle$

Without adjustable parameters :

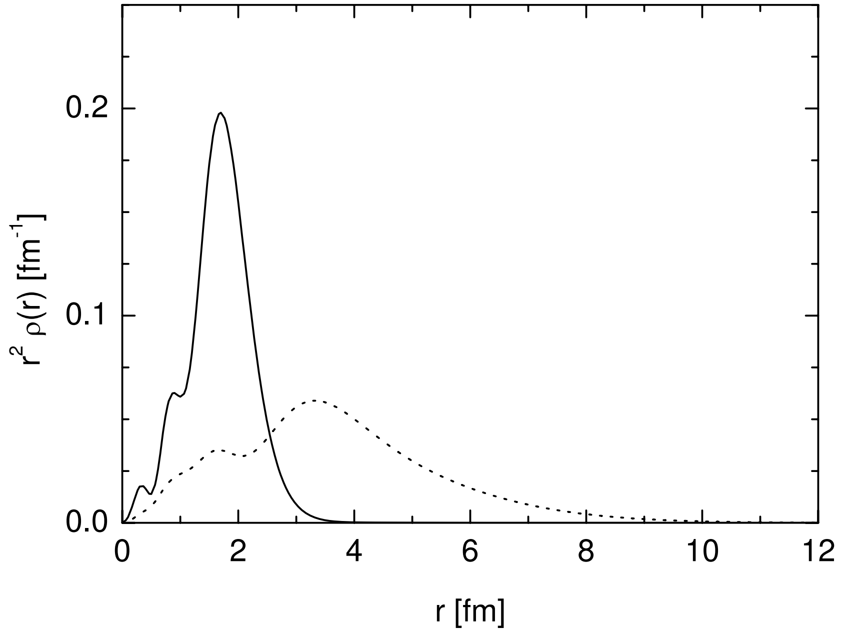
$$\begin{aligned} {}^{12}C : \quad (E_{O_2^+} - E_{3\alpha}) &= \text{Theory} + 0.50 \text{ MeV} \\ &\quad \text{Exp.} + 0.38 \text{ MeV} \\ {}^{16}O : \quad (E_{O_5^+} - E_{4\alpha}) &= \text{Theory} - 0.70 \text{ MeV} \\ &\quad \text{Exp.} - 0.44 \text{ MeV} \end{aligned}$$

r.m.s. of O_2^+ in ^{12}C 3.83 fm

groundstate 2.40 fm

$$\frac{V_{O_2^+}}{V_{O_1^+}} \sim 3-4$$

Density :

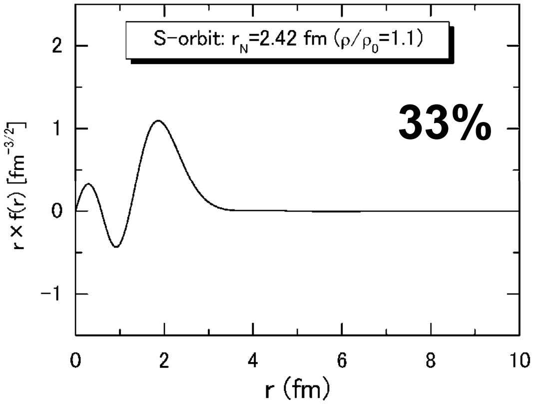


	$Exp.$	$Cal.$
$M(O_2^+ \rightarrow O_1^+)$	$5.4 \pm 0.2 fm^2$	6.7
$R_{rms}(O_1^+)$	$2.43 fm$	2.40
$R_{rms}(O_2^+)$		3.47

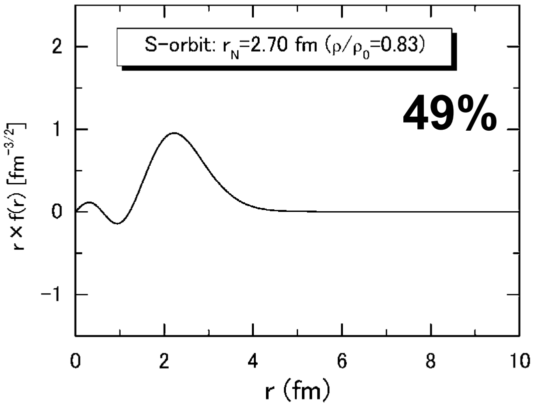
Radial behavior of *S*-wave α orbit vs. R_{rms}

R_{rms}=2.43 fm → 4.84 fm (ρ/ρ_0=1.1 → **0.14)**

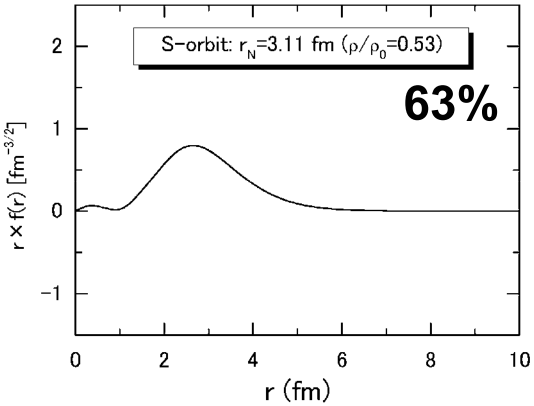
R_{rms}=2.43 fm (ρ/ρ_0=1.1)



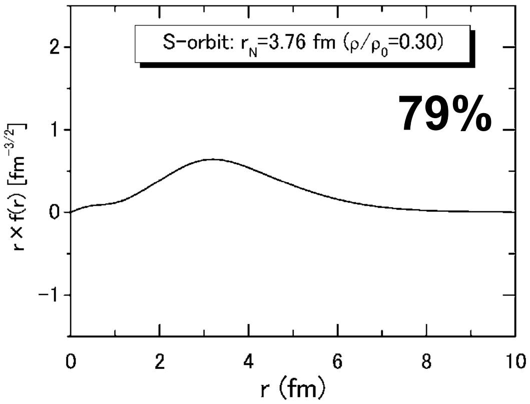
R_{rms}=2.70 fm (ρ/ρ_0=0.83)



R_{rms}=3.11 fm (ρ/ρ_0=0.53)



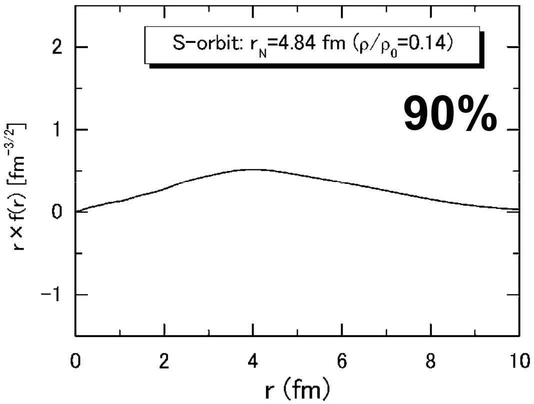
R_{rms}=3.76 fm (ρ/ρ_0=0.30)

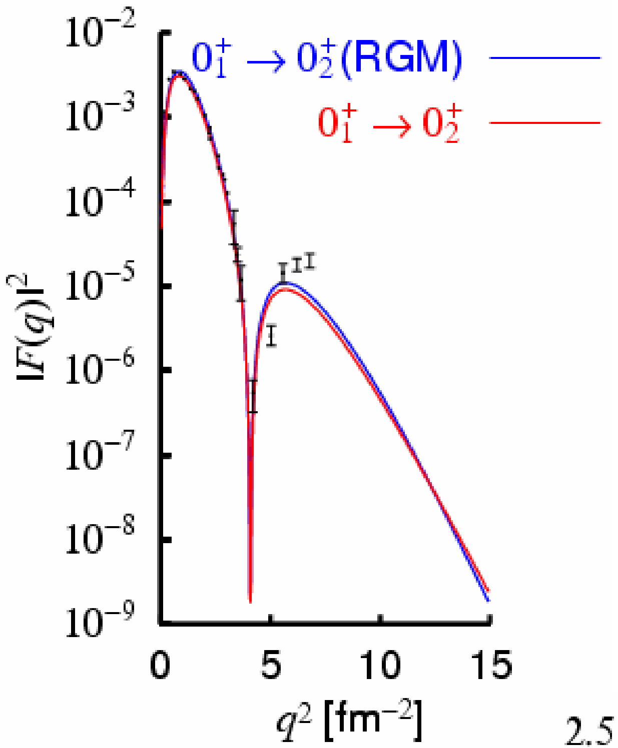


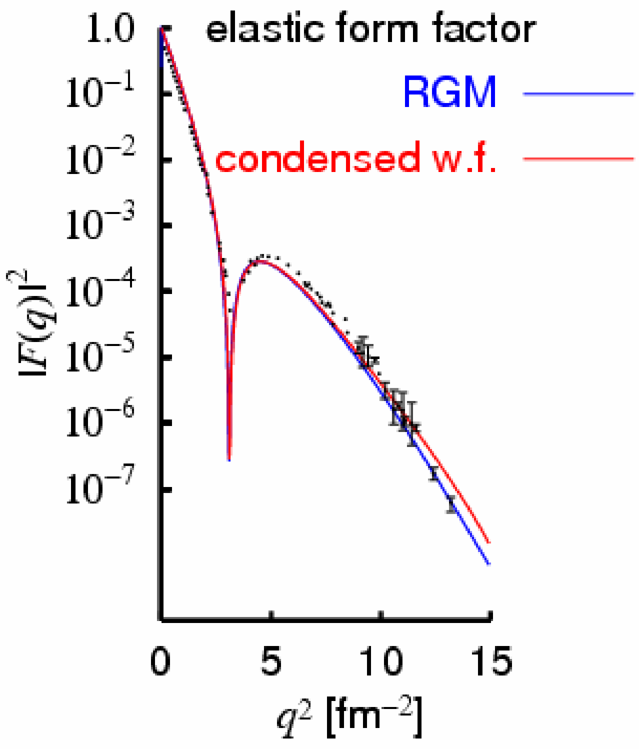
Increasing R_{rms}, we see smooth change from 2*S* to 0*S* orbit.

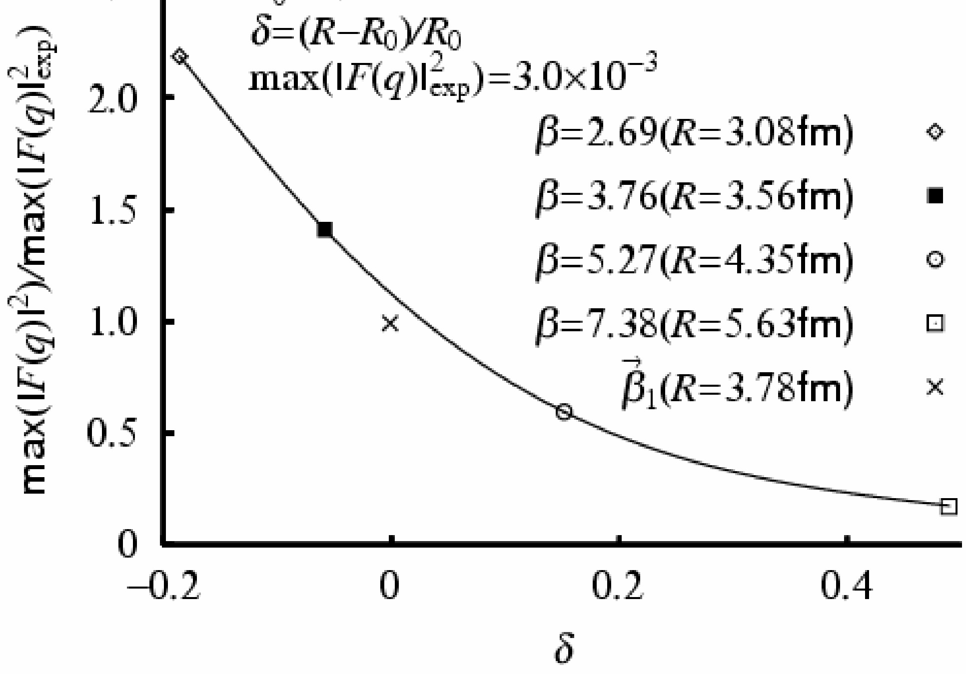
Yamada & Schuck, EPJA 2005

R_{rms}=4.84 fm (ρ/ρ_0=0.14)









R : r.m.s. radius

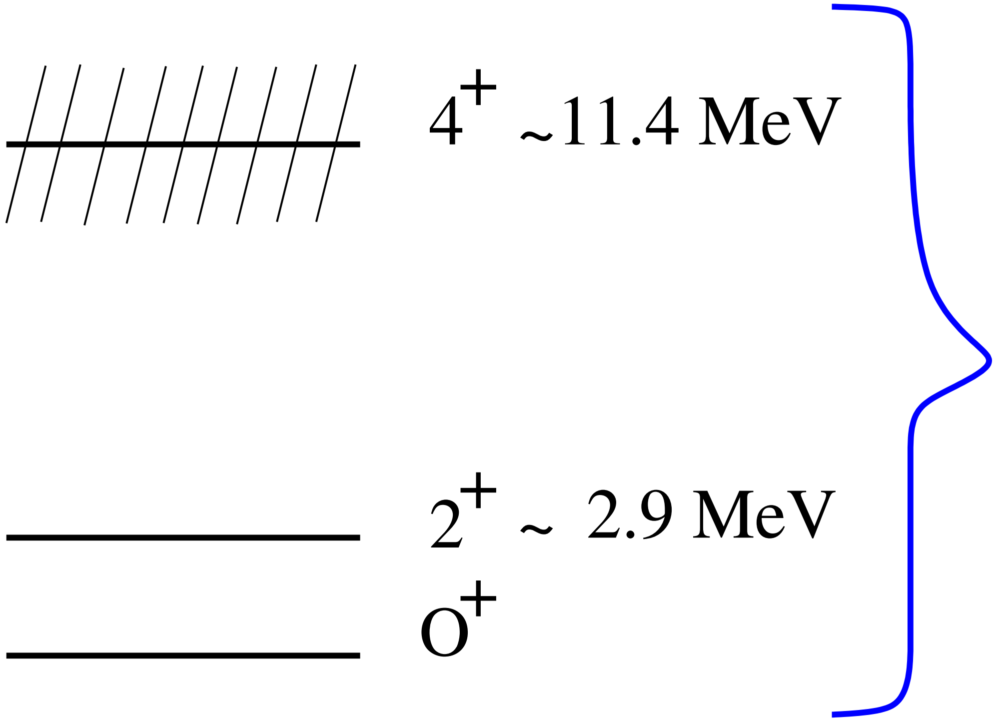
$\vec{\beta}_1 := (\beta_x = \beta_y,\ \beta_z)$
$= (5.27\ \mathrm{fm}, 1.37\ \mathrm{fm})$

Some more numbers :

		Theory	Exp.
^{12}C :	O_1^+	-89.52	-92.16
	O_2^+	-81.79	-84.51
		7.73	7.65

	^{8}Be	^{12}C	^{16}O	^{20}Ne	
	2α	3α	4α	5α	
Threshold states $E - E_{\text{thresh.}}$	O_1^+	O_2^+	O_3^+	O_4^+	theory
	-0.17	0.50	-0.7	1.8	
	O_1^+	O_2^+	O_5^+	?	experimental
	0.09	0.38	-0.44	?	

Spectrum of ^{8}Be :



4^{+} ~11.4 MeV

2^{+} ~ 2.9 MeV

0^{+}

Fully reproduced

^{12}C : Second excited 2^{+} : 2_2^{+}

It has been discovered recently by Itoh *et al.*
2.6 MeV above 3 α rhreshold
Width ~ 1 MeV : resonance in continuum

Theory : We start with deformed α condensate state :

$$\Phi_{n\alpha} \propto \mathcal{A} \prod_{i=1}^{n} \exp\left\{ -\frac{2\,X_{ix}^2}{B_x^2} - \frac{2\,X_{iy}^2}{B_y^2} - \frac{2\,X_{iz}^2}{B_z^2} \right\} \Phi_{\alpha_i}$$

Then projection on good angular momentum

Then Hill Wheeler or GCM
For width : ACCC method

	Position	Width
Experiment :	2.6 ± 0.3 MeV	1.0 ± 0.3 MeV
Theory :	2.1 MeV	0.64 MeV

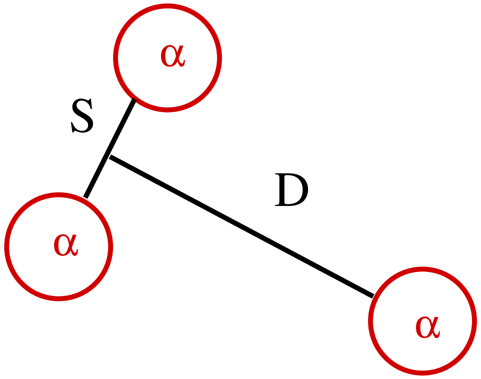
With in error bars !

RMS : $4.43\ fm$

$$\frac{V_{2_2^+}}{V_{O_1^+}} \sim 8 \quad !!$$

α – halo !

Internal structure :



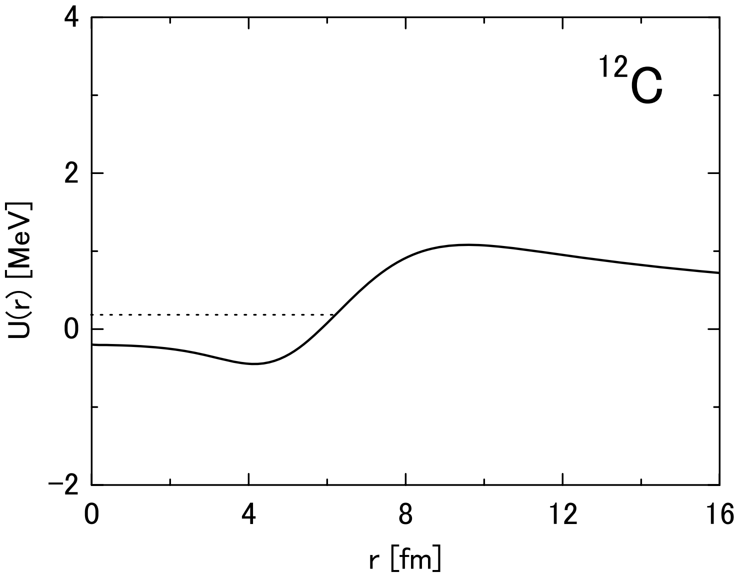
Extremely dilute 3α state

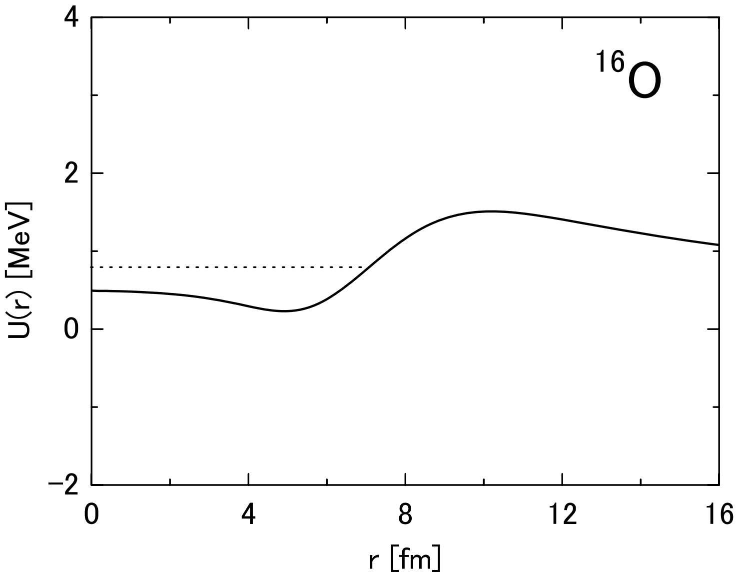
Suggests a pure Boson picture $|\phi_0\rangle = b_0^+ b_0^+ b_0^+ \ldots |vac\rangle$

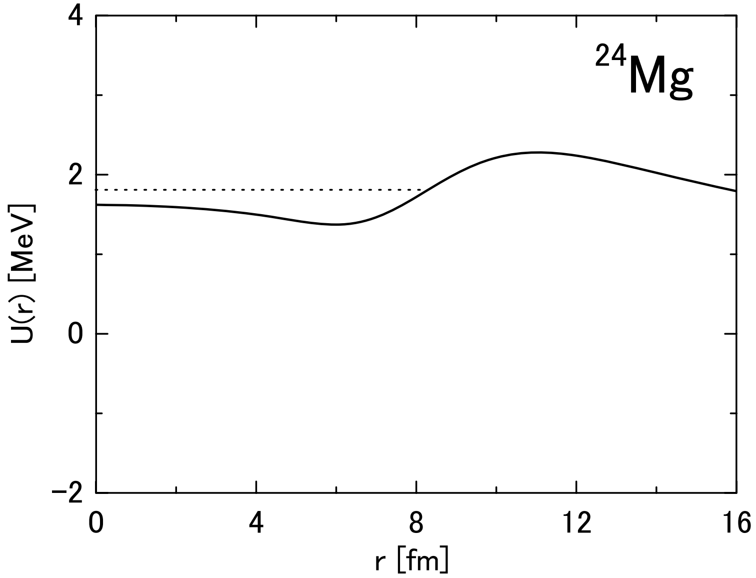
Hartree – Fock (Gross Pitaevsky eq)
for ideal bosons (α' s) :

$$\left[-\frac{\hbar}{2m_\alpha}\Delta + N\int d^3r' v(\vec{r}-\vec{r'})|\phi_0(\vec{r'})|^2\right]\phi_0(\vec{r}) = \epsilon_0\phi_0(\vec{r})$$

effective α – α + Coulomb
T. Yamada







Estimate for maximum number

$$\mathrm{N}^{\alpha}_{limit} \simeq 10 \qquad \Rightarrow \qquad {}^{40}\mathrm{Ca}^{**}$$

BUT

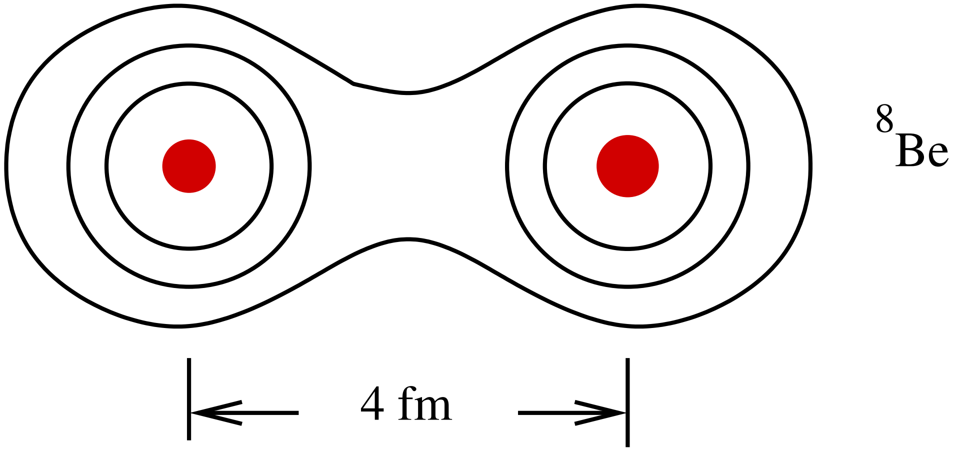
Some neutrons can stabilise

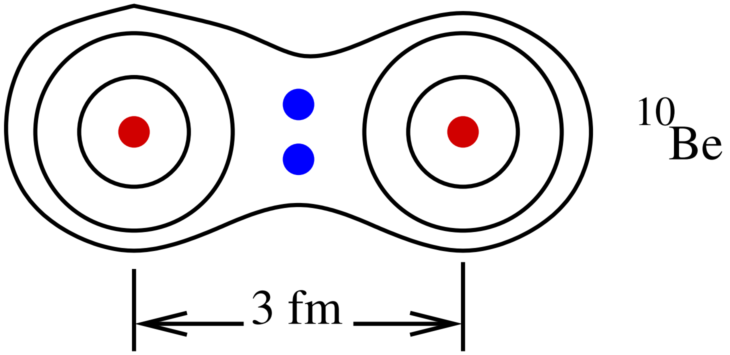
^{8}Be unbound, ^{9}Be bound (2.5 MeV)

^{10}Be strongly bound !

May be 20 - 30 α's

possible !





Boson occupancy :

α-particle density matrix :

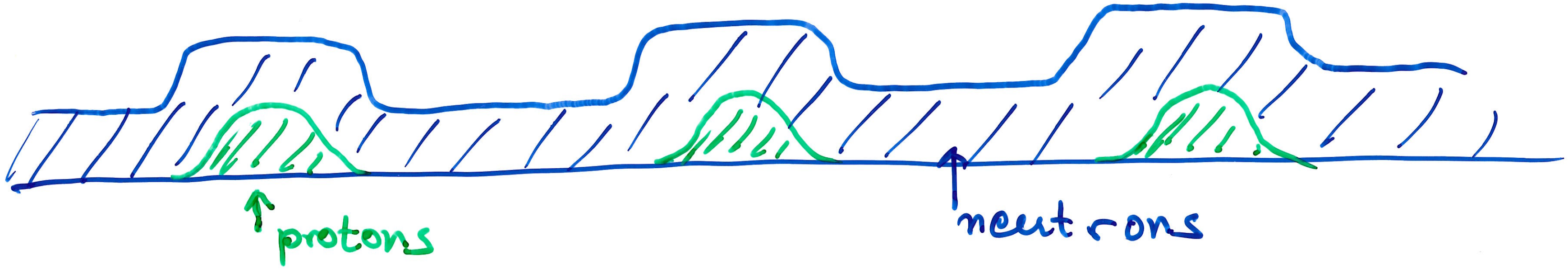
$$\rho_\alpha(\vec{R}, \vec{R}'), \quad \vec{R} \ : \text{ c.m. of } \alpha$$

Diagonalization :

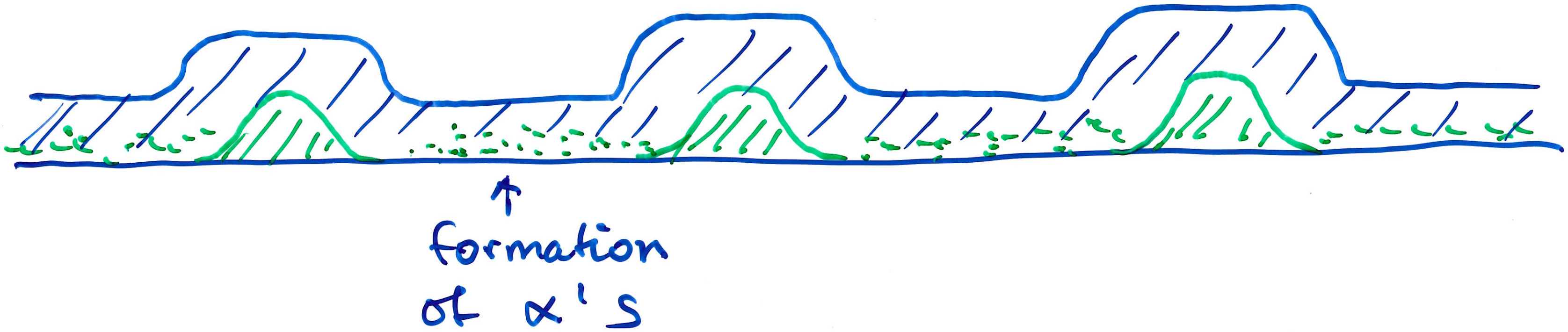
$$^{12}C: \quad O_2^+ \qquad 70\% \quad \text{S-wave occupancy}$$

Alpha particle condensation in crust of neutron stars ?

T=0



T>0



Conclusions

- Strong indications that extended α-condensates exist, α-condensate-halos

- Adding some neutron glue, condensates may become very large : ten's of α's ! Highly excited but long life time !

- Indra : $^{36}Ar + {}^{36}Ni \Rightarrow 9\alpha$
  (M.F. Rivet, B. Borderie)

- α-energies → wavefcts − Chimera

- Why α, not ^{16}O, ^{40}Ca
  α very inert and compact, almost ideal Boson, 20 MeV first excitation
  May be ^{48}Cr → 3 × ^{16}O

- Ultimate question :
  How to prove α-particle superfluidity ?

- Eventually important in nuclear astrophysics : collapsing massive stars
- Nuclei immersed in a gas of neutrons + some protons at finite $T \rightarrow$ Can lead to gas of α-particles
- may be α-particle condensate
- change of equation of state
- change of neutrino absorption
- influence on collapse scenario

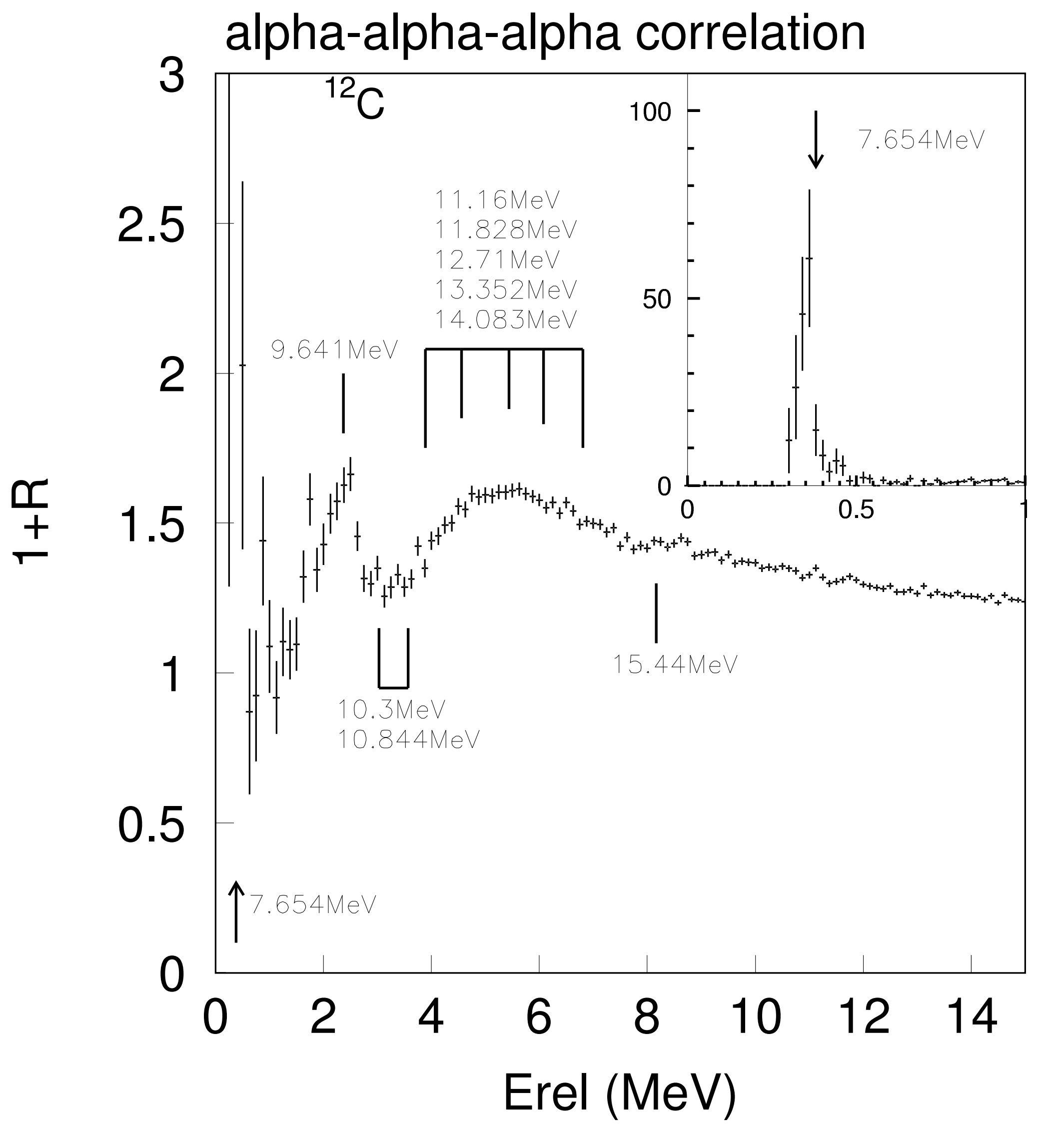
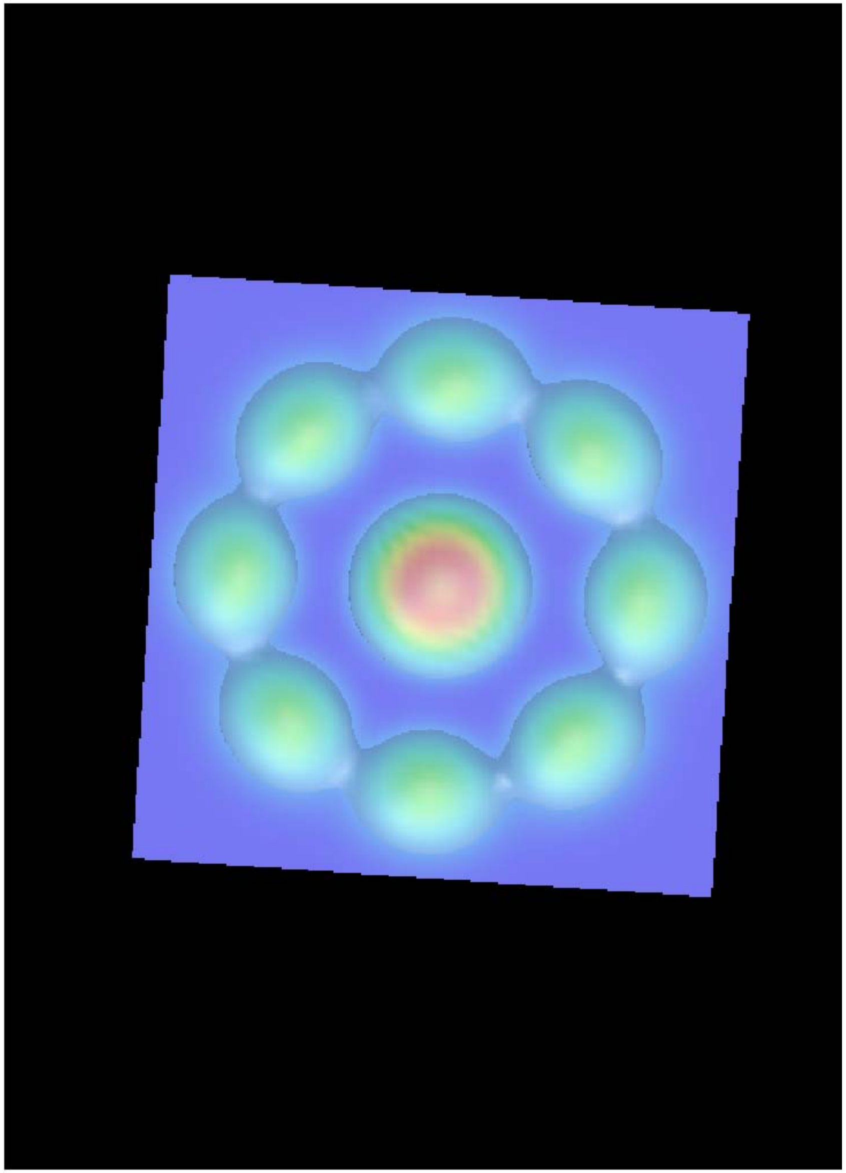
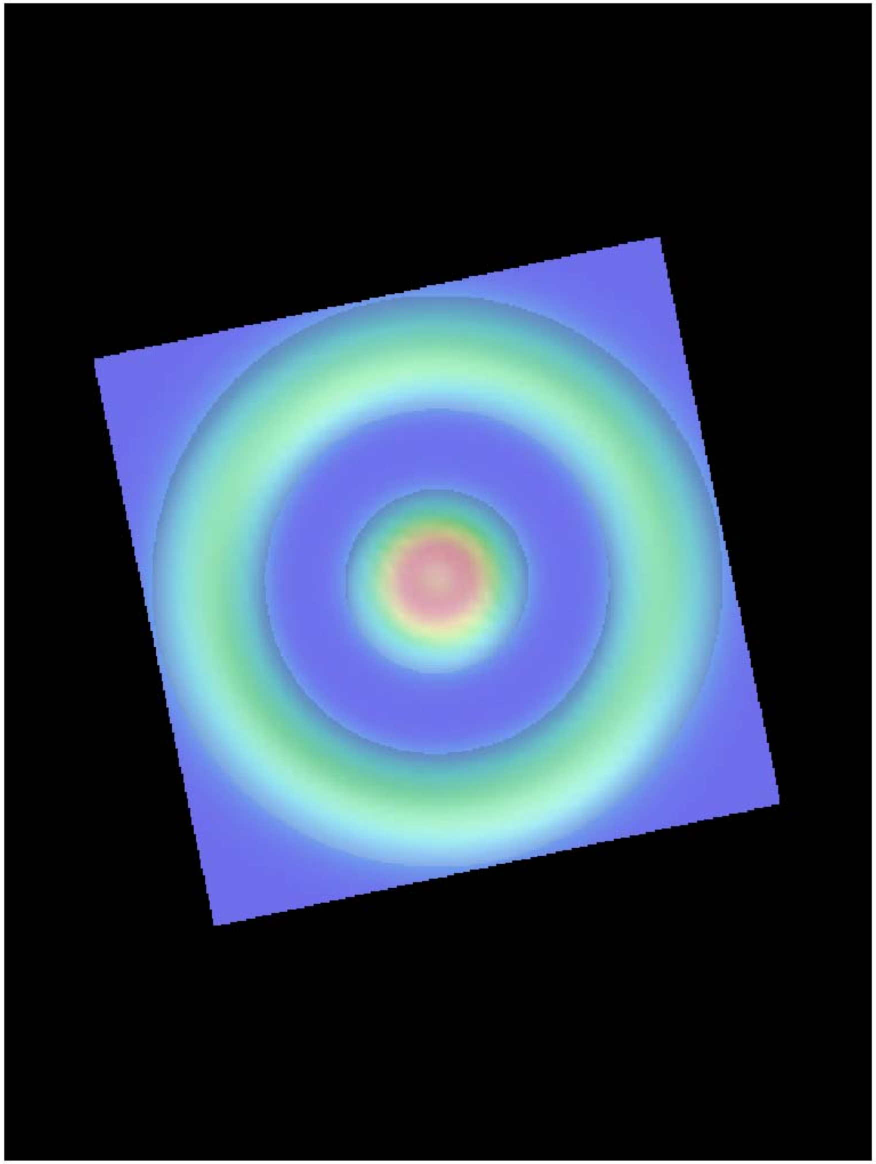
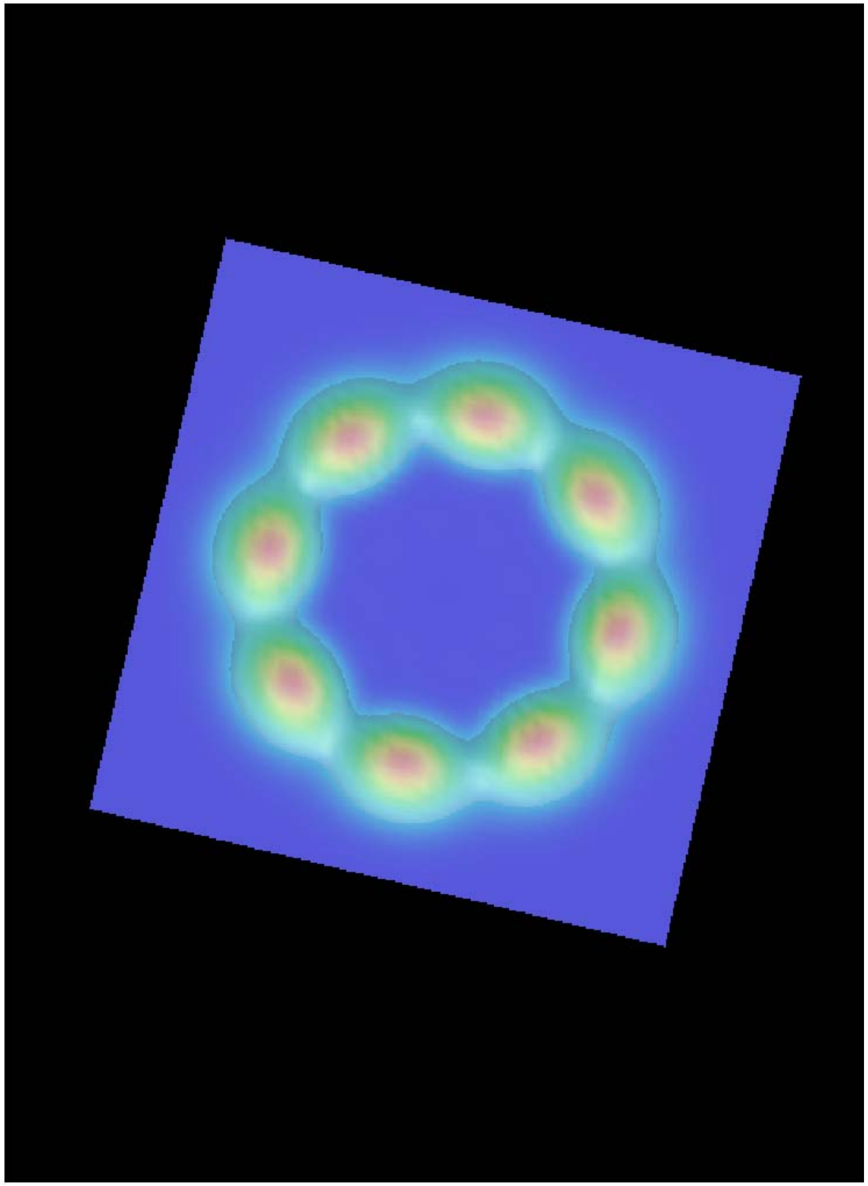
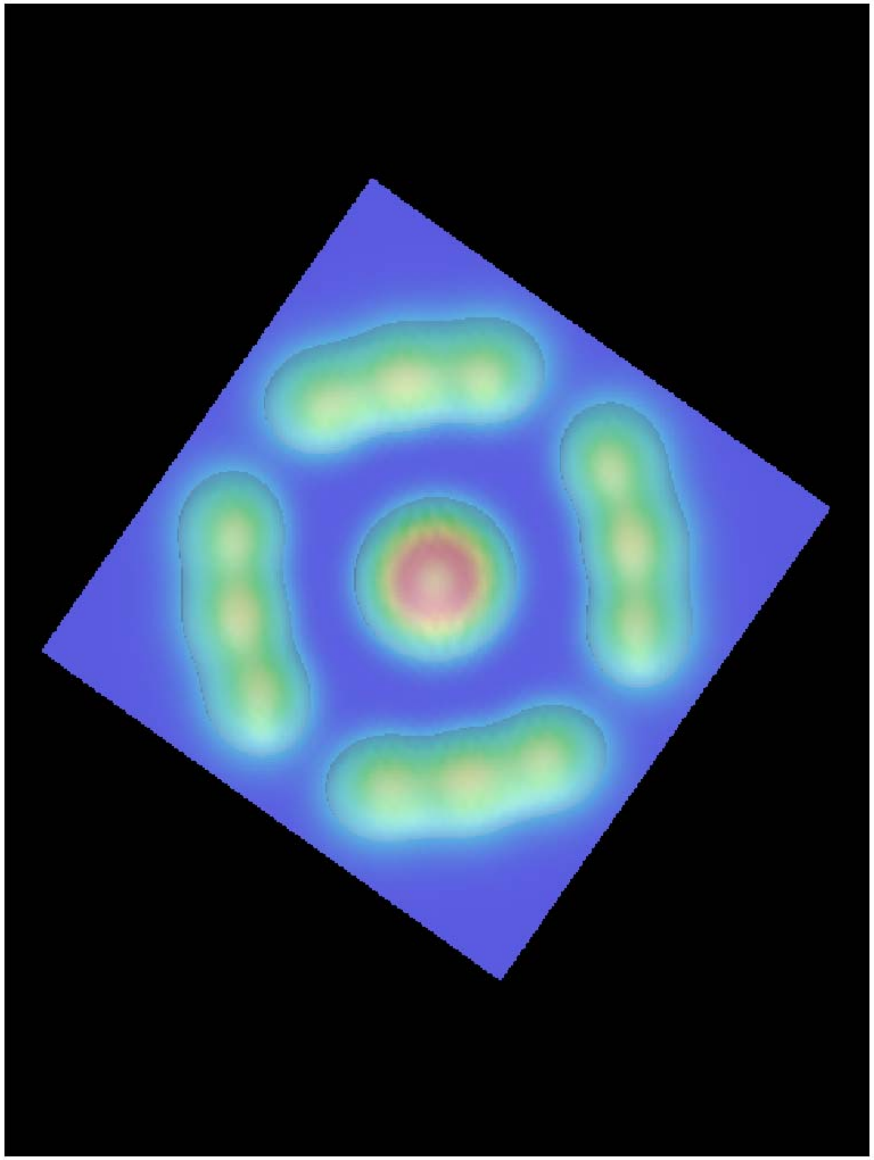


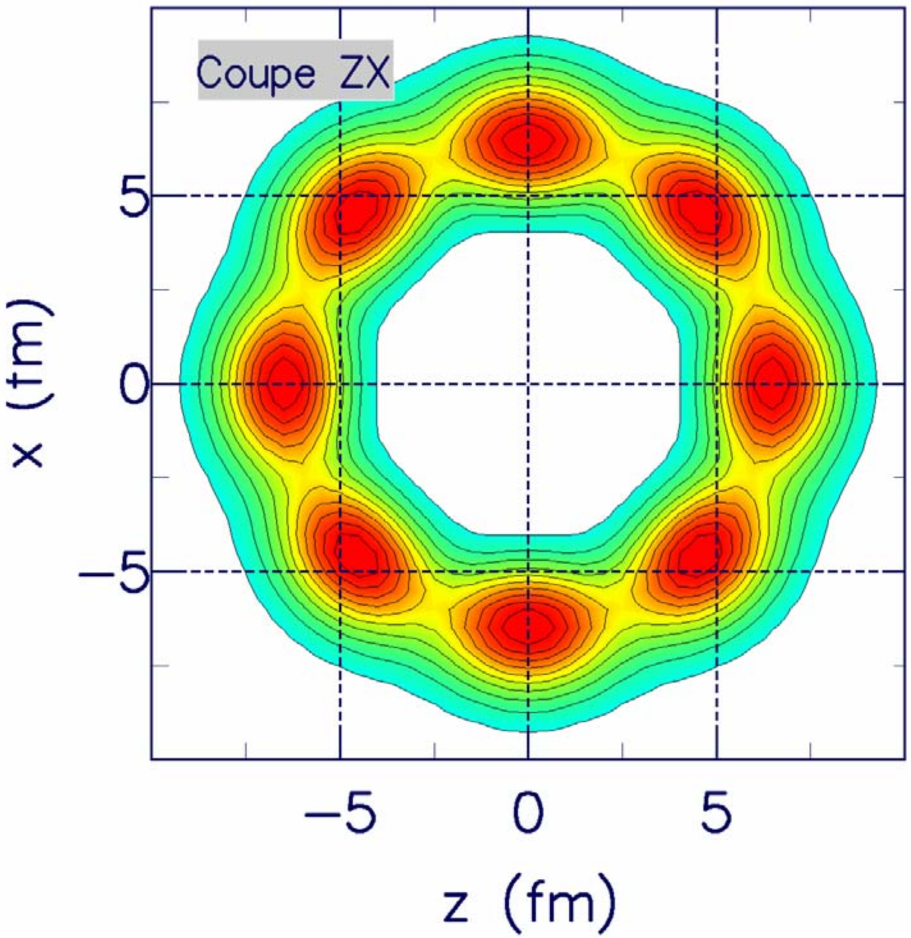
Figure 5.27: The α-α-α correlation function is shown. Resonances from the excited states of ^{12}C are labelled with the first peak seen more clearly in the inner upright panel.



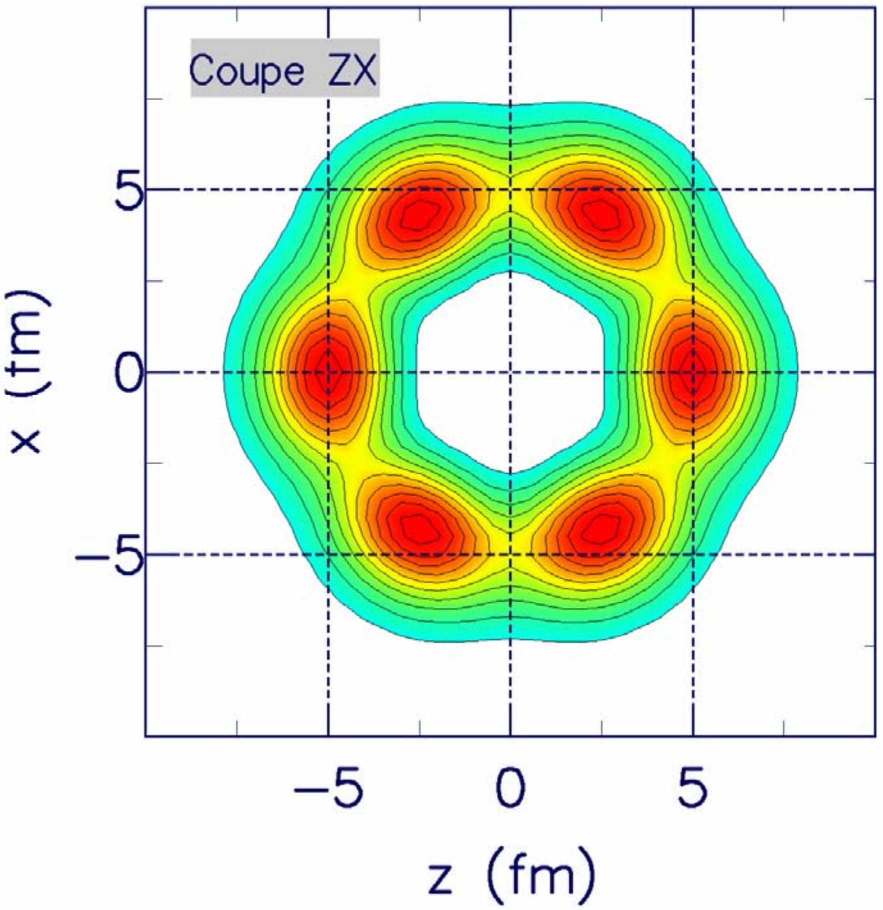








^{32}**S**



^{24}**Mg**

^{20}Ne