Inelastic scattering calculations with HF or HF+RPA target state description using D1S Gogny force

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Protons spectrum

$^{54}\text{Fe}(p,p')^{54}\text{Fe}^*$  \( E_{\text{inc}} = 62 \text{ MeV} \)

Direct reactions

Pre-equilibrium

Evaporation

Compound nucleus

Time

$^\text{54}\text{Fe}(p,p')^\ast$ $^\text{54}\text{Fe}^*$
Motivations
To improve predictive power of existing models

Ingredients:
- two-body interaction
- target state description

approximations
Adjustable parameters

Quantum pre-equilibrium models
Direct reaction models

Test
Inelastic scattering calculations with HF or HF+RPA target state
description using D1S Gogny force

- Direct reactions: microscopic calculations for
  - Elastic proton scattering
  - Inelastic proton scattering on discrete states.
- Pre-equilibrium calculations
  - One-step calculations $^{90}$Zr(p,p$'$) $^{90}$Zr
  - Brief review of two-steps calculations
- Conclusions and perspectives
NA elastic scattering: construction of the optical potential

Starting point:
two-body effective interaction representing the interaction between the projectile and the target nucleons

\[ \hat{V}_{\text{eff}}(1, 2) = \frac{1}{2} \sum_{\alpha \beta k k'} \langle \alpha k' | V_{\text{eff}}(1, 2) | \beta k \rangle a_\alpha^+ a_\beta a_{k'}^+ a_k \]

Elastic scattering process:

\[ |0\rangle \otimes |k\rangle \rightarrow |0\rangle \otimes |k'\rangle \quad \text{Target ground state} \]

\[ \hat{U}(1) = \langle 0 | \hat{V}_{\text{eff}}(1, 2) | 0 \rangle = \left\{ \sum_{\alpha \beta k k'} \langle \alpha k' | V_{\text{eff}}(1, 2) | \beta k \rangle \langle 0 | a_\alpha^+ a_\beta | 0 \rangle \right\} a_{k'}^+ a_k \]

\[ \hat{U}(1) = \sum_{k, k'} \left[ \begin{array}{c}
\begin{array}{c}
V_{\text{eff}} \\
U_L
\end{array}
\end{array} \right] + \left[ \begin{array}{c}
\begin{array}{c}
V_{\text{eff}} \\
U_{NL}
\end{array}
\end{array} \right] \]
\[ \hat{U}(1) = \langle 0|\hat{V}_{\text{eff}}(1, 2)|0\rangle = \sum_{\alpha\beta k k'} \langle \alpha k' | V_{\text{eff}}(1, 2) | \beta k \rangle \langle 0 | a_\alpha^+ a_\beta | 0 \rangle \left\{ a_{k'}^+ a_k \right\} \]

**Melbourne g-matrix**
Medium effect: density dependant. Central, spin-orbit and tensor terms. microscopic.

Double-closed shell nuclei:

Hartree-Fock with D1S interaction:
- Mean field theory

\[ |0\rangle \simeq |HF\rangle \]
\[ \langle HF|a^+_h a_{h'|HF} = \delta_{h,h'} \]

Random Phase Approximation (RPA) with D1S interaction:
- Correlated ground state

\[ |0\rangle \simeq |\tilde{0}\rangle \]
\[ \langle \tilde{0}|a^+_\alpha a_{\beta|\tilde{0}} = \rho_{\beta\alpha} \]

\[ \hat{U}(1) = \langle 0|\hat{V}_{\text{eff}}(1,2)|0\rangle = \sum_{\alpha\beta kk'} \langle \alpha k'|V_{\text{eff}}(1,2)|\beta k\rangle \langle 0|a^+_\alpha a_{\beta}|0\rangle a^+_k a_k \]
Elastic scattering

$^{208}\text{Pb} \,(p,p)^{208}\text{Pb}$

Target description:

- HF
- RPA

M. Dupuis, S. Karataglidis, E. Bauge, J.P. Delaroche et D. Gogny
Elastic scattering

$^{208}\text{Pb} \ (p,p)^{208}\text{Pb}$

Analysing power:

Target description:
- HF
- RPA
Elastic Scattering

$^{208}\text{Pb} \ (p,p)^{208}\text{Pb}$

Target description:

- HF
- RPA
Elastic scattering

Target description:
- HF
- RPA

Graphs showing the scattering cross-sections for different nuclei and energies.
Inelastic Scattering to discrete states:

Born expansion of the transition amplitude:

\[ T_{f \leftarrow i} = \langle \chi^{(-)}(k_f), n | V_{\text{eff}} + V_{\text{eff}} G^{(+)} V_{\text{eff}} + V_{\text{eff}} G^{(+)} V_{\text{eff}} G^{(+)} V_{\text{eff}} + \ldots | \tilde{0}, \chi^{(+)}(k_i) \rangle \]

\[ \frac{d\sigma(k_i, k_f)}{d\Omega_f} \sim \left| \langle \chi^{(-)}(k_f), n | V_{\text{eff}} | \tilde{0}, \chi^{(+)}(k_i) \rangle \right|^2 \]

\[ |n\rangle = \Theta_n^+ |\tilde{0}\rangle = \sum_{ph} \left( X_{ph}^n a_p^+ a_n - Y_{ph}^n a_p^+ a_n \right) |\tilde{0}\rangle \]

Coherent calculation of both elastic and inelastic scattering: ground and excited states = RPA states.

Description of collective states

The same interaction is used to generate distorted waves and for residual interaction: Melbourne g-matrix
Inelastic scattering

\[ ^{208}\text{Pb} (p,p')^{208}\text{Pb} \]

DWBA calculations with RPA excitations

Particle-hole excitations

RPA excitations
Inelastic scattering

$^{208}\text{Pb} \ (p,p')^{208}\text{Pb}$

Analysing powers

DWBA calculations with RPA excitations
Inelastic scattering
\( ^{208}\text{Pb} \ (p,p') \ ^{208}\text{Pb} \)

DWBA calculation with RPA excitations
Inelastic protons scattering

Others double-closed shell nuclei

A lot of excitations cannot be described with only the RPA method

Examples: $^{16}$O et $^{40}$Ca

Low energy excitations with positive parity

$^{16}$O $^3_-$

$^{40}$Ca $^{15}_-$

$^{48}$Ca $^{12}_+$

Configuration mixing

2p-2h ... excitations

DWBA calculations with RPA excitations
Protons inelastic scattering on $^{90}\text{Zr}$

**Double differential cross-section:**

$$\frac{d^2\sigma^{(1)}}{d\Omega_f dE_f} \sim \frac{1}{\Delta} \int_{E_f - \frac{\Delta}{2}}^{E_f + \frac{\Delta}{2}} dE \sum_n f_n(E_i, E) \left| \langle \chi^-(k_E), n | V_{\text{eff}} | 0, \chi^+(k_i) \rangle \right|^2$$

**Experimental resolution**

**Lorentzian**

$$\Gamma_n = \frac{1}{2} \frac{\Gamma_n}{(E - E_i - E_n + \delta_n)^2 + \frac{\Gamma_n^2}{4}}$$

**Energy shift**

$$E'_n = E_n - \delta_n + i \frac{\Gamma_n}{2}$$

**Damping width**

One-ph or one-boson excitations are coupled to more complicated states: 2p-2h …
$^{90}\text{Zr} (p,p') ^{90}\text{Zr}^*$

$E_{\text{inc}} = 120 \text{ MeV}$

$E_{\text{out}} = 110 \text{ MeV}$

one-step calculations with:

- particle–hole excitations
- or
- boson (RPA) excitations

$E_x =$

A.A. COWLEY et al. (1991)
$^{90}\text{Zr} (p,p') ^{90}\text{Zr}^*$

$E_{\text{inc}} = 120 \text{ MeV}$

$E_{\text{out}} = 106 \text{ MeV}$

one–step calculations with:

- particle–hole excitations
- boson (RPA) excitations

A.A. Cowley et al. (1991)
$^{90}\text{Zr} \ (p,p') \ ^{90}\text{Zr}^*$

$E_{\text{inc}} = 120 \ \text{MeV}$

$E_{\text{out}} = 100 \ \text{MeV}$

one-step calculations with:

- particle-hole excitations
- or
- boson (RPA) excitations

- A.A.COWLEY et al. (1991)
\[ ^{90}\text{Zr} \ (p,p') \ ^{90}\text{Zr}^* \]

First order calculations

\[ \sum \] Particle-hole excitations

\[ \sum \] RPA excitations

\[ E_{\text{inc}} = 120 \text{ MeV} \]
\[ E_{\text{out}} = 40 \text{ MeV} \]
\[ E^* = 80 \text{ MeV} \]

\[ E_{\text{out}} = 50 \text{ MeV} \]
\[ E^* = 70 \text{ MeV} \]

\[ E_{\text{out}} = 60 \text{ MeV} \]
\[ E^* = 60 \text{ MeV} \]

\[ E_{\text{out}} = 70 \text{ MeV} \]
\[ E^* = 50 \text{ MeV} \]
Motivations
To improve predictive power of existing models

Ingredients:
- two-body interaction
- target state description

approximations

Test

Quantum pre-equilibrium models

Reliable

Direct reaction models

Adjustable parameters

Test
Second order transition amplitude and FKK pre-equilibrium model

\[ T_{f\leftarrow i} \simeq \langle \chi^{(-)}(k_f), f | V_{\text{eff}} + V_{\text{eff}} G^{(+)} V_{\text{eff}} | 0, \chi^{(+)}(k_i) \rangle = T_{f\leftarrow i}^{\text{DWBA}} + T_{f\leftarrow i}^{(2)} \]

\[ T_{f\leftarrow i}^{(2)} = \sum_n \int \frac{dk}{(2\pi)^3} \langle \chi^{(-)}(k_f), f | V_{\text{eff}} | \chi^{(+)}(k), n \rangle \]

\[ \chi \left( \frac{1}{E - E_n - E_k + i\epsilon} \right \langle \hat{\chi}^{(+)}(k), n | V_{\text{eff}} | 0, \chi^{(+)}(k_i) \rangle \]
Usual approximations in quantum pre-equilibrium models

On shell approximation

\[
\frac{1}{E - E_n - E_k + i\epsilon} = -i\pi\delta(E - E_n - E_k) + P \left\{ \frac{1}{E - E_n - E_k} \right\}
\]

Target states description: particle-hole excitations

Intermediate states

\[
|n\rangle = |1p1h\rangle = A_{ph}^+(jm)|HF\rangle = \left[ a_p^+ \otimes a_h \right]^j_m |HF\rangle
\]

Final states

\[
|f\rangle = |2p2h\rangle = \left[ A_{p_1h_1}^+(j_1m_1) \otimes A_{p_2h_2}^+(j_2m_2) \right]^J_M |HF\rangle
\]
Double differential second step cross section:

\[
\frac{d^2 \sigma^{(2)}(k_i, k_f)}{d\Omega_f dE_f} \sim \frac{1}{\Delta} \int_{E_f - \frac{\Delta}{2}}^{E_f + \frac{\Delta}{2}} dE \sum_f \frac{1}{2} \frac{\Gamma_f}{(E - E_i - E_f)^2 + \frac{\Gamma^2}{4}} \times
\]

\[
\sum_n \langle \chi^-(k_E), f | V_{\text{eff}} | n, \chi^+(k) \rangle \langle \hat{\chi}^+(k), n | V_{\text{eff}} | 0, \chi^+(k_i) \rangle
\]

Test of FKK (Feshbach, Kerman, Koonin) model assumptions
H. Feshbach assumption:

\[ \langle \chi^{(-)}(k_E), f|V_{\text{eff}}|n, \chi^{(+)}(k) \rangle \langle \chi^{(+)}(k), n|V_{\text{eff}}|0, \chi^{(+)}(k_i) \rangle \]

Energy averaging  

\[ \langle \chi^{(-)}(k_E), f|V_{\text{eff}}|n, \chi^{(+)}(k) \rangle \langle \chi^{(-)}(k), n|V_{\text{eff}}|0, \chi^{(+)}(k_i) \rangle \]

Justification not clear,  
but always used !!!
$^{90}\text{Zr} \ (p,p') \ ^{90}\text{Zr}^*$

$E_{\text{inc}} = 120 \text{ MeV}$

$E_{\text{out}} = 100 \text{ MeV}$

First order

"exact" calculation

Second order:

- FKK approximation

Factor $\sim 20$ !!
Second order cross-section is overestimated: possible causes

• Melbourne G-matrix already includes some second order processes:  
  Double counting?

• On energy shell approximation:  
  No convergence without it!!
Conclusions and perspectives: direct reactions

• Melbourne G-matrix + RPA/D1S: good description of direct reactions:
  - medium energy elastic scattering
  - inelastic protons scattering on discrete states of the target nuclei

  No adjustment !!!

Extension to spherical nuclei with open shell(s):

QRPA description of target states.

Configuration mixing theory

The effective interaction (Melbourne G-matrix) is density dependant: rearrangement corrections.
Conclusions and perspectives: pre-equilibrium

• Pre-equilibrium calculations for (p,p’) reaction:
  • First step is well described, sufficient for low energy transfers between the projectile and the target (<20 MeV).

• Second order calculations:
  - Feshbach assumption is not reliable
  - Second order cross-section is overestimated

Solutions ?
Conclusions and perspectives: pre-equilibrium

• Inclusion of real continuum state in particle-hole excitations:
  \[ a^+_k a_h \]

  More realistic

  Angular distribution of second particle emission

• Second order calculations of \(^{90}\text{Zr}(p,p')^{90}\text{Zr}\) with RPA excited state: could avoid double counting.

  Convergence of off-shell components?

  Third step calculations to test the convergence of the Born series expansion
Collaborators

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